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Non-tidal changes in the Earth's rate of rotation as deduced from medieval eclipse observations

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Summary. A series of 60 careful timings of both solar and lunar eclipses measured between A.D. 829 and 1019 by Muslim astronomers is analysed to investigate the extent of non-tidal variations in the Earth's rate of rotation on the millennial time-scale. Many of these observations have never previously been analysed for this purpose. Our results indicate that at approximately A.D. 950 – the average epoch of the data – the length of the mean solar day was 11.6 ± 0.6 milliseconds shorter than the standard day of 86400 SI seconds. This corresponds to an average rate of lengthening of the day over the past 1000 years or so of 1.37 ± 0.07 milliseconds per century (ms/cy). The observed rate of increase is fully 1 ms/cy less than the value computed on the assumption of tidal friction alone (2.4 ms/cy). Further, it is considerably less than the mean rate of increase over the past 2500 years (2.0 ms/cy) – as derived by Stephenson and Morrison (1984) from ancient Babylonian observations. Hence the medieval data provide firm evidence for substantial long-term variations in the length of the day of non-tidal origin.

Key words: Earth's rotation – lunar eclipses – solar eclipses

1. Introduction

Changes in the rate of rotation of the Earth over the past 2700 years, as revealed by a variety of astronomical observations, were studied in some detail by Stephenson and Morrison (1984). [This reference will subsequently be abbreviated to SM]. These authors deduced that the mean rate of lengthening of the day between about 500 B.C. and. A.D. 1000 was 2.4 ms/cy, approximately the theoretical tidal figure. However, over the last millennium the observed mean was found to have diminished to approximately 1.4 ms/cy, equivalent to a major increase in the non-tidal component.

For the telescopic period, by far the most important data source utilised by SM was occultations of stars by the Moon and these were used to map fluctuations in the Earth's rotation in detail. However, throughout the pre-telescopic period, the investigation by SM concentrated exclusively on solar and lunar eclipses; these were judged to be the most useful ancient and medieval data. The bulk of the pre-telescopic observations analysed by SM covered two quite distinct time-interals (roughly from 700 B.C. to A.D. 100 and again between A.D 700 and 1250). Lacking many suitable eclipse reports at other periods, it proved impossible to map the observed changes in the length of the day in detail. Analysis of timed measurements formed the main feature of the investigation by SM. The number of medieval data of this kind was considerably less than for the corresponding ancient data, thus somewhat weakening their conclusions. In view of the importance of better understanding of the geophysical mechanisms responsible for long-term changes in the Earth's rotation, further investigation of both ancient and medieval records of eclipses and other events (e.g. occultations of stars and planets by the Moon) is desirable. In this paper we attempt to refine the solution of SM by analysing in detail an expanded set of medieval timings of both solar and lunar eclipses. We also include summaries of the observations used.

Before undertaking the present investigation, we compiled a list of 60 medieval timings of solar and lunar eclipse contacts measured by Muslim astronomers. These observations, dating from between A.D. 829 and 1019, were largely recorded at Baghdad and Cairo. They are among the most accurate of extant medieval eclipse timings from any part of the world. For several centuries before and afterwards, very few carefully timed observations are preserved in history, thus emphasising the importance of the present data set. Some of the records analysed here were investigated by SM but we also include a substantial number of additional observations.

2. Medieval Islamic timings of lunar and solar eclipses

Measurements of this type first attracted the attention of Newcomb more than a century ago (1878). In turn, Newcomb's paper formed the main source of medieval eclipse timings used by SM. Newcomb published and analysed a series of timings of both lunar and solar eclipses dating from between A.D. 829 and 1004. These were measured by astronomers first at Baghdad (until A.D. 933) and later at Cairo (from A.D. 977). The observations are recorded in an astronomical text by the great Cairo astronomer ibn Yūnus (1008), who died in A.D. 1009. This work, which is in Arabic, was dedicated to Caliph al-Ḥākim, and is entitled *al-Zīj al-Kabīr al-Ḥākimī*. (The term *Zīj* means an astronomical handbook with tables, while *Kabīr* means large). ibn Yūnus assembled observations of as many as 28 lunar and solar eclipses, several of which he had witnessed himself.

In his investigation, Newcomb (1878) utilised a translation from Arabic into French by Caussin (1804) of the observational

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Send offprint requests to: F.R. Stephenson accounts recorded by ibn Yūnus. These reports are contained in the Leiden manuscript Or. 143, which Caussin consulted. This manuscript contains only the first 22 chapters of the original text of ibn Yūnus cited above, the eclipse records, together with other observational material) being contained in chapters 4, 5 and 6 of this work (King, 1976). Caussin noted several textual errors but his translation still contains a few mistakes. We have therefore retranslated the observational records direct from the printed Arabic text published by Caussin along with his French translation.

We have supplemented the ibn Yūnus material with a number of eclipse timings recorded by two other medieval Muslim astronomers: al-Battānī (who lived between A.D. 850 and 929) and al-Bīrūnī (A.D. 973–1048). Observations dating from between A.D. 883 and 901 are contained in an Arabic work by al-Battānī (c. 910) which has been edited and translated into Latin by Nallino (1899). al-Bīrūnī (1025 and 1030), in two works also written in Arabic, cites several eclipse timings recorded between A.D. 1002 and 1019. The earlier text has been translated into English by Ali (1967). In all cases we have consulted the printed Arabic version directly. Newton (1970) investigated most of these observations but he relied on the available translations into a European language. Further his work was done at a time when the orbital acceleration of the Moon (due to tides) was still only poorly determined. Hence his results have lost much of their relevance today.

For each of the eclipses reported by ibn Yūnus, al-Battānī and al-Bīrūnī the place of observation is clearly expressed in the original text. The geographical co-ordinates of these various sites (in degrees and decimals) – together with the original place names and abbreviations used in subsequent tables in this paper – are summarised in Table 1. Dates present few difficulties; nearly all are given in terms of the Islamic lunar calendar. We have effected conversions to the Julian calendar using a computer program based on the tables of Freeman-Grenville (1977). Most dates so reduced agree within a day of that of a tabular eclipse – e.g. as listed by Oppolzer (1887). Occasionally, the discrepancy is larger (typically a full month), but such errors are easily rectified owing to the relative infrequency of eclipses visible at any one site.

Approximate predictions for both lunar and solar obscurations were made by the Muslim astronomers, so that usually they would have made advance preparations for observation. For example, we occasionally find statements such as the following by ibn Yūnus (relating to the solar eclipse of A.D. 977):

"In order to observe this eclipse, several scholars assembled at Qarafa (a district of Cairo) in the mosque of al-Maghribi. Everyone waited for the start of the eclipse . . ."

Table 1. Geographical co-ordinates of places of observation

Modern name	Original name	Abbrev.	N lat.	E long.
Antakya	Antakya	A	36°.20	36°.17
Baghdad	Baghdad	B	33°.33	44°.43
Cairo	al-Qahira	C	30°.05	31°.25
Ghazni	Ghazna	Gh	33°.55	68°.47
Gorgan	Jurjan	Go	36°.83	54°.48
Kunya-Urgench	Jurjaniyya	K	42°.30	59°.17
Raqqa	Raqqa	R	35°.95	39°.05

Of special interest in this connection are the planned joint observations of the lunar and solar eclipses of A.D. 901. These were observed at Antakya by al-Battānī and at Raqqa by an assistant.

As a result of such preparations, we might reasonably expect the beginning of an eclipse to be determined almost as carefully as the end. In two examples (A.D. 986 and 1004), the observers imply that they had missed first contact (possibly on account of intermittent cloud?). However, in each case they give an estimate of the true moment of contact, based on the proportion of the lunar or solar disc obscured when the eclipse became "noticeable to the view".

Most recorded timings relate to first or fourth contact (i.e. the beginning or end of an eclipse). Only in a single instance – in A.D. 854 – is the onset of totality (for a lunar eclipse) noted. However, the time of mid-eclipse is occasionally reported. This was sometimes based on direct observation at maximum phase. Less frequently, the reported time of mid-eclipse represented a mean of the results for first and fourth contact; unfortunately in the latter case the original observations are never given. Maximum phase is, of course, not usually a well-defined moment unless a large proportion of the Sun or Moon is obscured and Newcomb (1878) regarded such determinations as valueless. However, we have included observations of this kind in the present investigation; in our opinion, errors in the measurement of time are likely to be comparable with other observational errors.

Times of both lunar and solar eclipses were normally measured indirectly by the Muslim astronomers using altitude determinations; these were afterwards converted to local time by them with the aid of an astrolabe. Altitudes were taken either of the Sun (in the case of a solar eclipse), or of the Moon or a suitable bright star (for a lunar obscuration). Although most observations were made at only two locations – Baghdad and Cairo – over the almost 200-year time interval, a variety of instruments were used for altitude determinations. Hence it is not possible for us to make any correction for systematic instrumental errors. Occasionally it is reported that more than one observer made separate measurements of the same eclipse contact. For example in A.D. 923 we find the following remark by the Baghdad observer ibn Amajur, as reported by ibn Yūnus:

"Abū al-Ḥasan estimated that mid-eclipse (of the Sun) occured at 8 deg altitude, as I estimated myself".

al-Bīrūnī cites as many as four separate altitude determinations (each for a different star) made in Ghazni at the beginning of the lunar eclipse of A.D. 1019. (These observations were first analysed by Newton, 1972).

Altitude measurements at the various eclipse contacts are usually expressed to the nearest degree or half degree. At these fairly low latitudes (near 35 deg N), the altitude of a celestial body typically changes by approximately one degree in every 5 minutes unless it is near the meridian. Hence the above rounding errors will normally correspond to timing uncertainties of less than about 0.05 hour, although instrumental defects seem likely to increase these uncertainties appreciably. In many instances, the local time – as determined by an astronomer from his altitude measurement – accompanies the original observation. These times, expressed relative to either midnight, sunrise, noon or sunset, provide a useful check on the veracity of the altitude measurements as preserved in the printed Arabic texts. In several other cases, only the local time is recorded without stating how this was deduced. Such times are typically quoted to the nearest 0.1 or 0.2 hour but the actual precision is very variable. Time units may be either unequal (seasonal) hours, equal hours or "minutes of day", these units corresponding to 1/60th of a day (0.4 hour).

An unusual observation is reported in A.D. 990. The time of end for the lunar eclipse of that year is given as the "rising of the start of Aquarius", i.e. a point on the ecliptic whose longitude – relative to the equinox of data – was then close to 300 deg.

The various measurements which we have assembled are summarised in the first five columns of both Table 2 (solar eclipses) and Table 3 (lunar eclipses). The remaining columns of these tables contain the results of our computation, as discussed in Sect. 3 below. Columns 1 to 5 contain the following information:

column 1: a reference number.
column 2: the date on the Julian calendar (abbreviated to A.D. date).
column 3: an initial letter (or letters) representing the place of observation.
column 4: the appropriate eclipse contact.
column 5: a summary of the actual altitude or time determination.

If the reference number in column 1 is followed by the letter 'a', we have given an amended altitude reading in column 5 on account of a suspected textual error in the preceding entry (see below).

Observations of the two lunar eclipses in A.D. 1003 were made at Gorgan and the local times were reduced by al-Bīrūnī to the longitude of Ghazni. We have restored the times cited by al-Bīrūnī to Gorgan using his own rather precise results for the difference in longitude between the two cities (equivalent to 14.1 deg, compared with the true value of 13.99 deg).

In column 4, the numbers 1, 2 and 4 indicate respectively first contact, start of totality (for a lunar eclipse) and last contact. The letter 'm' denotes mid-eclipse as estimated directly whereas 'M' implies a mean calculated by the observer from his measurements of both 1st and 4th contact.

The abbreviations SR and SS in column 5 denote sunrise and sunset respectively. We have carefully checked Caussin's conversion of Arabic star names to their modern equivalents using a variety of published works. In reference number 52, we have removed any confusion over the identity of the Arabic star name *al-Ḥadī*; this definitely corresponds to α Aurigae, whereas Caussin had been somewhat doubtful as to its true identity. Certain records state that a measurement of altitude or time is only approximate. In other examples, an altitude or time is said to be either a little more or a little less than the figure quoted. Such remarks are indicated in column 5. Normally the text clearly states whether an altitude measurement was made relative to the E or W horizon. In default, this can usually be inferred from the text – e.g. by an allusion to the Moon or Sun rising or setting eclipsed – and we have indicated such inferences using parentheses.

The ibn Yūnus and al-Bīrūnī texts quote measurements using numerals based on the letters of the Arabic alphabet. This is similar to the practice of the ancient Greeks. Using such a script, the presence or absence of diacritical marks can lead to widely differing numbers. As discussed in Sect. 3, we have proposed, with varying degrees of confidence, five alternative readings (see reference numbers 32, 35, 47, 51 and 53 in Table 3). Only when a variant reading differs by more than 10 deg from the original

Table 2. Summary of medieval Islamic timings of solar eclipses

Ref.	A.D. date	Place	Contact	Measured alt. or time	UT (h)	ET (h)	ΔT (s)
01	829 Nov. 30	B	4	Alt. of Sun = 24° (E)	6.44	7.06	2240
02	866 Jun. 16	B	1	> 1/3 h (uneq.) after noon	> 9.42	10.09	< 2420
03	866 Jun. 16	B	m	7 h 26 m (uneq.) after SR	10.74	11.43	2500
04	866 Jun. 16	B	4	8 h 30 m (uneq.) after SR	12.01	12.69	2480
05	891 Aug. 08	R	m	1 h (uneq.) after noon	10.59	11.06	1680
06	901 Jan. 23	A	m	∼3 h 40 m (eq.) before noon	∼6.17	6.63	∼1640
07	901 Jan. 23	R	m	< 3 h 30 m (eq.) before noon	> 6.15	6.66	< 1840
08	923 Nov. 11	B	m	Alt. of Sun = 8° E	4.37	4.91	1960
09	923 Nov. 11	B	4	Alt. of Sun = 20° E	5.53	5.98	1600
10	928 Aug. 18	B	4	Alt. of Sun = 11.9° E	3.50	4.01	1820
11	977 Dec. 13	C	1	Alt. of Sun = 15.5° (E)	6.29	6.80	1820
12	977 Dec. 13	C	4	Alt. of Sun = 33.3° (E)	8.60	9.15	1980
13	978 Jun. 08	C	1	Alt. of Sun ∼ 56° (W)	∼12.37	12.73	∼1280
14	978 Jun. 08	C	4	Alt. of Sun ∼ 26° (W)	∼14.70	15.26	∼2020
15	979 May. 28	C	1	Alt. of Sun = 6.5° (W)	16.21	16.62	1460
16	985 Jul. 20	C	1	Alt. of Sun ∼ 23° (W)	14.93	15.19	940
17	985 Jul. 20	C	4	Alt. of Sun = 6° (W)	16.30	16.51	760
18	993 Aug. 20	C	1	Alt. of Sun = 27° E	5.60	6.16	2000
19	993 Aug. 20	C	m	Alt. of Sun = 45° E	7.01	7.40	1400
20	993 Aug. 20	C	4	Alt. of Sun = 60° E	8.28	8.72	1580
21	1004 Jan. 24	C	1	Alt. of Sun = 18.5° W	13.85	14.25	1440
22	1004 Jan. 24	C	m	Alt. of Sun = 5° W	15.05	15.37	1140

Table 3. Summary of medieval Islamic timings of lunar eclipses

Ref.	A.D. date	Place	Contact	Measured alt. or time	UT (h)	ET (h)	ΔT (s)
23	854 Feb. 16	B	1	∼10h 03m afternoon	∼19.35	20.22	∼3140
24	854 Aug. 12	B	1	Alt. of α Tau = 45.5°	0.01	0.71	2520
25	854 Aug. 12	B	2	Alt. of α CMi ∼ 22.5° E	∼1.53	1.91	∼1340
26	856 Jun. 22	B	1	Alt. of α Tau = 9.5° E	0.42	1.08	2360
27	883 Jul. 23	R	m	> 8h (eq.) after noon	> 17.47	17.82	< 1280
28	901 Aug. 02	A	m	∼15h 20m (eq.) after noon	∼0.98	1.16	∼640
29	901 Aug. 02	R	m	∼15h 35m (eq.) after noon	∼1.04	1.16	∼420
30	923 Jun. 01	B	m	1h 40m (eq.) after SS	17.78	18.12	1240
31	923 Jun. 01	B	4	Alt. of α Cyg = 29.5° E	18.94	19.50	2020
32	925 Apr. 11	B	1	Alt. of α Boo = 11° E	14.64	17.13	(9000)
32a	925 Apr. 11	B	1	Alt. of α Boo = 31° E	16.31	17.13	2960
33	925 Apr. 11	B	4	Alt. of α Lyr = 24° (E)	19.79	20.44	2340
34	927 Sep. 14	B	1	Alt. of α CMa = 31° E	0.91	1.73	2940
35	929 Jan. 27	B	1	Alt. of α Boo = 18° E	20.10	21.77	(6000)
35a	929 Jan. 27	B	1	Alt. of α Boo = 33° E	21.32	21.77	1600
36	933 Nov. 05	B	1	Alt. of α Boo = 15° E	1.36	1.99	2260
37	979 May. 14	C	4	∼1.2 h (eq.) after SS	∼17.85	18.31	∼1660
38	979 Nov. 06	C	1	Alt. of Moon = 64.5° E	20.08	20.61	1900
39	979 Nov. 06	C	4	Alt. of Moon = 65° W	23.30	23.67	1320
40	980 May. 03	C	1	Alt. of Moon = 47.7°?	(Error)	22.77	?
41	980 May. 03	C	4	∼0.6 h (eq.) before SR	∼2.49	3.07	∼2080
42	981 Apr. 22	C	1	Alt. of Moon ∼ 21° (W)	∼1.39	1.97	∼2080
43	981 Apr. 22	C	4	∼0.25 h before SR	∼3.01	3.67	∼2360
44	981 Oct. 16	C	1	Alt. of Moon ∼ 24° (W)	∼2.11	2.54	∼1540
45	983 Mar. 02	C	1	Alt. of Moon = 66°	∼22.13	22.61	∼1720
46	983 Mar. 02	C	4	Alt. of Moon = 35.8° (W)	1.55	1.87	1160
47	986 Dec. 19	C	1	Alt. of Moon = 50.5° W	0.82	3.07	(8100)
47a	986 Dec. 19	C	1	Alt. of Moon = 30.5° W	2.33	3.07	2680
48	990 Apr. 12	C	1	Alt. of Moon = 38° (E)	19.70	20.69	3540
49	990 Apr. 12	C	4	Rising of start of Aqu.	23.17	23.85	2460
50	1001 Sep. 05	C	4	2 h (uneq.) after SS	18.01	18.17	600
51	1002 Mar. 01	C	1	Alt. of α Boo = 12° E	18.45	22.15	(13300)
51a	1002 Mar. 01	C	1	Alt. of α Boo = 52° E	21.61	22.15	1960
52	1002 Mar. 01	C	1	Alt. of α Aur = 14° W	21.67	22.15	1740
53	1002 Mar. 01	C	4	Alt. of α Boo = 35° (E)	4.59	1.58	(−10800)
53a	1002 Mar. 01	C	4	Alt. of α Boo = 75° E	1.49	1.58	320
54	1003 Feb. 19	Go	M	6.73 h (eq.) after noon	15.34	15.75	1480
55	1003 Aug. 15	Go	M	11.63 h (eq.) after noon	20.03	20.20	600
56	1004 Jul. 04	K	M	14.61 h (eq.) after noon	22.72	23.45	2620
57	1019 Sep. 17	Gh	1	Alt. of α Aur < 66° E	< 21.62	22.13	> 1820
58	1019 Sep. 17	Gh	1	Alt. of α CMa = 17° E	21.65	22.13	1720
59	1019 Sep. 17	Gh	1	Alt. of α CMi = 22° E	21.63	22.13	1800
60	1019 Sep. 17	Gh	1	Alt. of α Tau = 63° E	21.69	22.13	1580

have we suggested a restoration. However, in the case of one obvious textual error (ref 40) we have been unable to offer a viable alternative reading. On this occasion, the altitude of the Moon at the start of the lunar eclipse is given as 47 deg 40 min. As Newcomb (1878) noted, this altitude is impossible; the *meridian* altitude of the Moon was no more than 42.5 deg.

Two careful descriptions of central or near-central eclipses (occurring in A.D. 1004 and 873) are reported by ibn Yūnus and al-Bīrūnī and we include these observations in the present investigation. It is well established that an untimed record of a large solar eclipse which specifically asserts that at a certain place either (i) the total or annular phase was witnessed or (ii) the eclipse remained incomplete can be of considerable value in the study of changes in the Earth's rate of rotation. Unlike many medieval reports of this kind, the following two accounts have the attraction that the place of observation is clearly stated.

Further, both reports are by astronomers and in each case we have an unusually careful description of the eclipsed Sun at maximum obscuration.

ibn Yūnus cites an observation which he made at Cairo on a date corresponding to A.D. 1004 Jan 24: "The Sun was eclipsed until what remained of it resembled the crescent Moon on the first day of the month; I estimated the magnitude of the eclipse as 11 digits . . . ". The quoted magnitude corresponds to 11 twelfths of the solar diameter, although this is likely to have been underestimated on account of the effects of irradiation. However, it seems clear that the central phase of this generally annular eclipse was not witnessed at Cairo. The various stages of this eclipse were also timed – see Table 2, refs. 21 and 22 – but the observed magnitude can be analysed quite independently of these measurements.

al-Bīrūnī (1030) quotes an account of an annular eclipse which occurred a century before his own time. This was seen on a date equivalent to 28 July in A.D. 873 at Neyshabur (original name Nisāpūr: lat 36.22 deg N, long 58.82 deg E). al-Bīrūnī writes that an astronomer named Abū al-Abbās al-Irānshahrī carefully observed the eclipse at this location early in the morning. "He mentioned that the body of the Moon was in the middle of the body of the Sun so that the light from the remaining portion of the Sun surrounded it uneclipsed; and it is clear from this that the diameter of the Sun exceeds in view that of the Moon". Attention was drawn to this observation by Goldstein (1979). His translation is in good agreement with our own.

3. Analysis of the observations

In analysing the observations discussed in Sect. 2, we have assumed that the only unknown parameter is the clock error ΔT arising from irregularities in the Earth's rate of rotation. This quantity may be defined at any particular epoch as the difference between TDT (Terrestrial Dynamical Time) and UT (Universal Time). TDT is the time-argument of current lunar and planetary ephemerides; for the present purpose, this may be regarded as identical to its predecessor ET (Ephemeris Time). UT is measured by the Earth's rotation. We have computed solar coordinates using Newcomb's theory (1895). We have verified by trial comparisons that co-ordinates derived in this way for medieval dates are in excellent agreement with those deduced from the Jet Propulsion Laboratory Integrated Ephemeris DE 102 (Newhall et al., 1983). Calculations of lunar position were made using the ephemeris $j=2$ (IAU, 1968), but with an addition to the mean lunar longitude L of

$$\Delta L = -1.54 + 2.33T - 1.78T^2 \text{ arcsec.}$$

This correction incorporates a lunar acceleration ($\dot{n}$) of -26 arcsec/cy^2. For details, see the investigation by Morrison (1979). In the above expression, ΔT is measured in Julian centuries from the epoch 1900.0. The selected value of $\dot{n}$ of -26 arcsec/cy^2 is in close accord with the result of -25.3 ± 1.2 derived by Dickey et al. (1984) from lunar laser ranging.

For each individual observation listed in Tables 2 and 3 we first calculated the equivalent UT from the recorded altitude or local time measurement. Our results, for convenience rounded to the nearest 0.01 hour, are summarised in column 6 of Tables 2 and 3. The TDT of each eclipse contact, given in column 7 of these tables, was calculated using specially designed computer programs. Finally in column 8 we give the appropriate ΔT values (TDT – UT), rounded to the nearest 20 sec.

In the case of a solar eclipse, the calculated TDT of any contact varies considerably with the location of the observer on the Earth's surface. For a given station, the TDT of a particular solar eclipse phase must be computed using the ephemeris longitude of the site rather than its geographic longitude, the two meridians essentially differing by ΔT. Since our objective has been to deduce ΔT from each individual observation, we have used an iterative procedure to derive the appropriate values of TDT and ΔT. In the case of a lunar eclipse, the TDT of entry into or exit from the Earth's shadow is virtually independent of the observer's location so that the calculated value of TDT is independent of the UT.

For all solar eclipse timings, we have assumed geometrical contacts. The limited resolution of the human eye will mean a small delay in detecting the beginning of an eclipse and a slightly premature observation of its end. Fortunately, there is almost an equal number of first and fourth contact timings for solar eclipses in our list (Table 2) so that errors of this nature should roughly cancel. In making calculations for present-day lunar eclipses, it is customary to assume a small increment in the radius of the Earth's shadow (by 2%) in order to approximately allow for the effect of the terrestrial atmosphere (see e.g. Smart, 1977). We have followed this rule in the present investigation. The number of recorded first contacts for lunar eclipses is more than twice the number of last contacts but the balance is partly redressed by the substantial number of timings for mid-eclipse.

The mean of all the ΔT results in Tables 2 and 3 is around 2000 sec. There is a considerable scatter about this mean but few values deviate by more than 1000 sec from it. However, for observations numbered 32, 35, 47, 51 and 53 (all Table 3) deviations are clearly excessive (amounting to at least an hour) which suggests scribal mistakes rather than errors of measurement. In each case, we have given probable variant readings (reference numbers being followed by the letter a). The manuscript figures for 18 and 33 (ref 35), 50 and 30 (ref 47), and 12 and 52 (ref 51) are all readily confused depending on the presence or absence of diacritical points. Assumption of such textul errors leads to values of ΔT within the typical scatter range. A scribal error in reference 47 is evident since the text states that this lunar eclipse first "became sensible to the view" when the altitude of the Moon was 24 deg W and yet the altitude of the Moon at true contact was estimated at 50 deg 30 min W. The intervening time of about 2 hours would be more than half the duration of this partial obscuration. Hence we have assumed an initial lunar altitude of 30 deg 30 min W. A reading of 52 deg E in place of 12 deg E for the altitude of α Boo in reference 51 is confirmed by the separate measurement, made at the same contact, of 14 deg W for the altitude of α Aur; the revised figure gives a result for the UT of the observation in close agreement (discrepancy less than 4 minutes) with that deduced from the altitude of α Aur.

We have also suggested copying errors in references 32 and 53. In the former case we have proposed reading 31 deg in place of 11 deg for the altitude of α Boo. Although confusion between the numbers 11 and 31 is unusual in manuscripts, there is direct evidence in the same text in favour of this amendment. As well as the altitude measurement, the record also gives the local time (apparently deduced from this altitude) as 55 minutes (unequal) after sunset. At this local time, the altitude of α Boo would be close to 31 deg, hence justifying our correction. Finally, we have suggested a scribal mistake in reference 53, reading 75 deg for 35 deg for the altitude of α Boo at 4th contact. Such a copying error is again uncommon. However, comparison between the UT of last contact deduced on the assumption of an altitude of 35 deg for α Boo with that derived for first contact from the measurement of altitude in reference 52 would give an impossibly long duration for this lunar eclipse of some 7 hours. Replacement of 35 deg by 75 deg gives an acceptable duration of rather less than 4 hours, close to the computed duration.

In the subsequent analysis, where a measurement is reported as "a little less" or "a little more" than a certain value, we have assumed that value itself; it is not possible to ascertain just what these terms meant and fortunately remarks of this kind are very rare. Despite the non-uniformity of the data, we have decided to assign equal weights to all the observations listed in Tables 2 and 3 – including the five amended altitude readings just discussed. For such early data, weighting is highly subjective – and in any case the scatter in the ΔT values is much larger than might be expected from rounding errors alone.

In order for the solar eclipse of A.D. 873 to have been annular at Neyshabur, a value of ΔT somewhere in the range 1820 to 3750 sec is required at this date. For the solar eclipse of A.D. 1004 to have been partial at Cairo, either a value of ΔT less than 1770 sec or alternatively one greater than 1940 sec is necessary; intermediate values are excluded. These latter limits are so close together because the track of annularity across the Earth's surface in A.D. 1004 was unusually narrow.

4. Results and interpretation

Figure 1 shows a plot of the individual values of ΔT listed in Tables 2 and 3, solar eclipse observations being denoted by open circles and lunar eclipse observations by shaded circles. Also shown – by vertical lines – are the firm limits for ΔT required to produce annularity at Neyshabur in A.D. 873 and a partial (non-central) eclipse at Cairo in A.D. 1004; in the latter case there is only a narrow zone of avoidance. The computed straight line of best fit (least squares) to the timed data is indicated: this shows the expected decrease in ΔT with increasing year, due to the gradual lengthening of the day. However, a more physically meaningful result is obtained by considering a parabolic fit to the data as follows.

Lambeck (1980) and SM demonstrated that there are sound geophysical reasons for assuming a virtually steady lunar and solar tidal torque over the last few millennia. Sea-level changes on the continental margins during this period were probably too small to alter tidal dissipation appreciably, while the relevant astronomical parameters remained accurately static. Hence if non-tidal effects had been negligible, the rate of increase in the length of the day (LOD) over the historical period would be

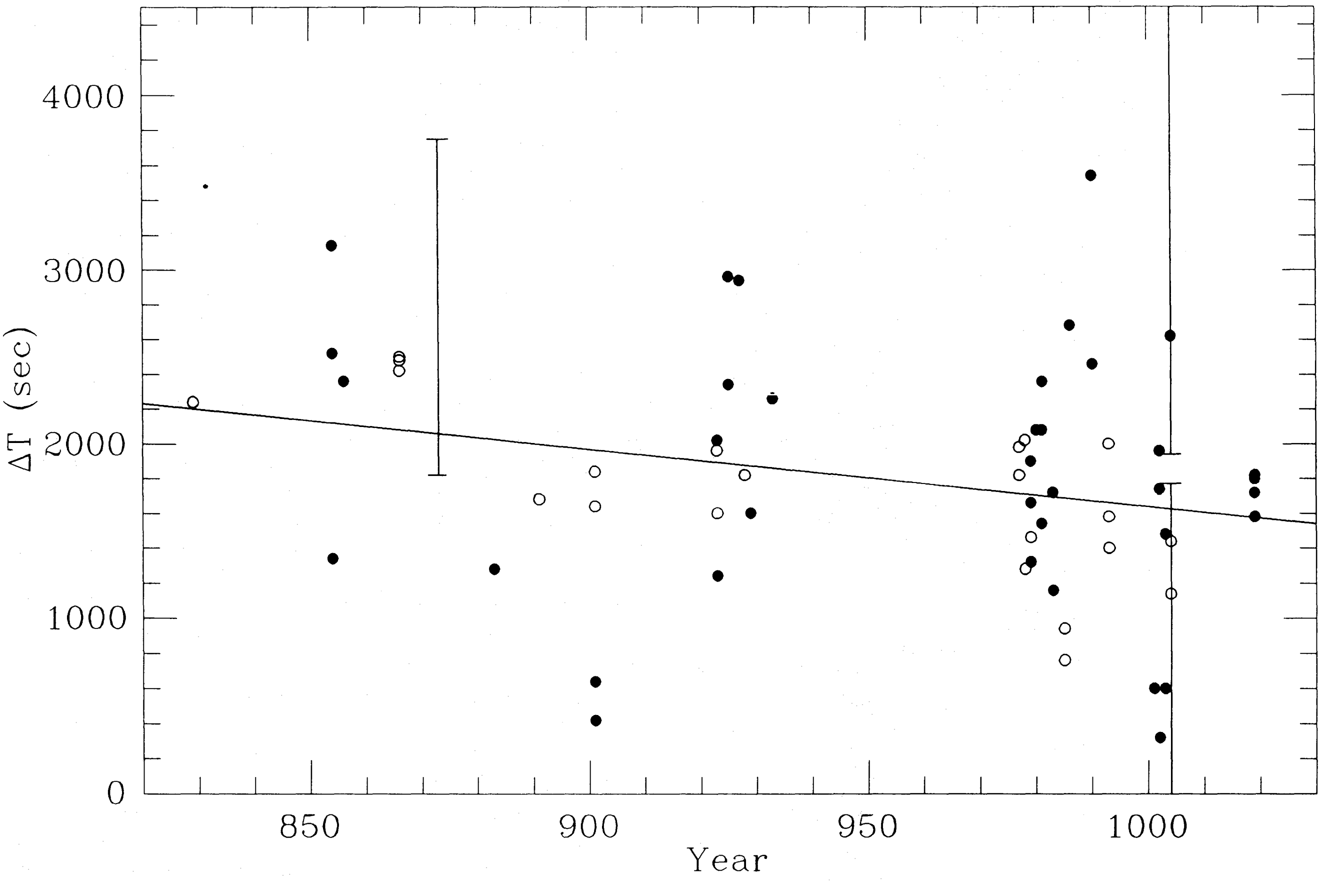

Fig. 1. ΔT values derived from medieval observations of solar and lunar eclipses in the period A.D. 829–1019. Open circles represent solar eclipse timings, shaded circles lunar eclipse timings. The vertical lines at A.D. 873 and 1004 represent the ΔT limits required to achieve annularity on these dates. The sloping straight line is the least squares fit to the data points

sensibly constant and the accumulated clock error ΔT would thus be parabolic. Any non-tidal effects are best studied relative to the mean tidal parabola, especially in view of the concentration of good medieval data over a very restricted time interval. SM showed that the parabola which best fits the ΔT results derived from telescopic observations has a minimum near A.D. 1800. This is then the approximate year at which the length of the UT day is equal to the fixed reference day of 86400 SI seconds. Because the value of ΔT was itself close to zero around A.D. 1800, it is convenient to fit the medieval data approximately by a parabola of form

$$\Delta T = ct^2,$$

where c is a constant and t is time measured in centuries from A.D. 1800.

The mean value of c calculated from the ΔT results obtained for the solar eclipses listed in Table 2 (23.3 ± 1.1 sec/cy^2) is in good agreement with that derived from the lunar eclipses in Table 3 (26.0 ± 1.8). Taking the mean of all 59 results (for both solar and lunar eclipses) gives

$$c = 25.0 \pm 1.2 \text{ sec/cy}^2.$$

A value of c in this range is supported both by the report of the annular phase at Neyshabur in A.D. 873 and the observation of an incomplete eclipse at Cairo in A.D. 1004. However, the narrow zone of exclusion in the latter year does not enable a unique solution to be made independently.

The observed value for c of 25.0 sec/cy^2 falls far short of the theoretical tidal figure of 43.8 sec/cy^2, which follows from the investigation by Lambeck (1980). [Lambeck expressed his result in a slightly different form]. It is also substantially less than the average value of c over the last 2500 years as derived from ancient observations alone; the analysis by SM of a large number of Babylonian timings of lunar eclipses (of mean epoch 390 B.C.) gave a very accurate result of 32.5 ± 0.5 sec/cy^2.

In Fig. 2 we have plotted the same medieval data points as in Fig. 1. However, we have also shown (by a full line) the mean parabola through these points, derived above, having the equation $\Delta T = 25.0\, t^2$ sec. For comparison, the tidal parabola ($c = 43.8$) and long-term Babylonian mean ($c = 32.5$) are also depicted (broken lines). The discrepancy between the medieval parabola and the other two curves is marked, emphasising the inadequacy of the assumption of (a) an Earth's rotational history in which only tidal friction played a significant role or (b) a constant non-tidal torque over the past 2500 years.

Changes in the LOD (Δ LOD) derived from medieval eclipse timings are shown in Fig. 3 relative to the reference day of 86400 SI seconds. This diagram, which covers the period from A.D. 800 to the present day, also includes the so-called "decade

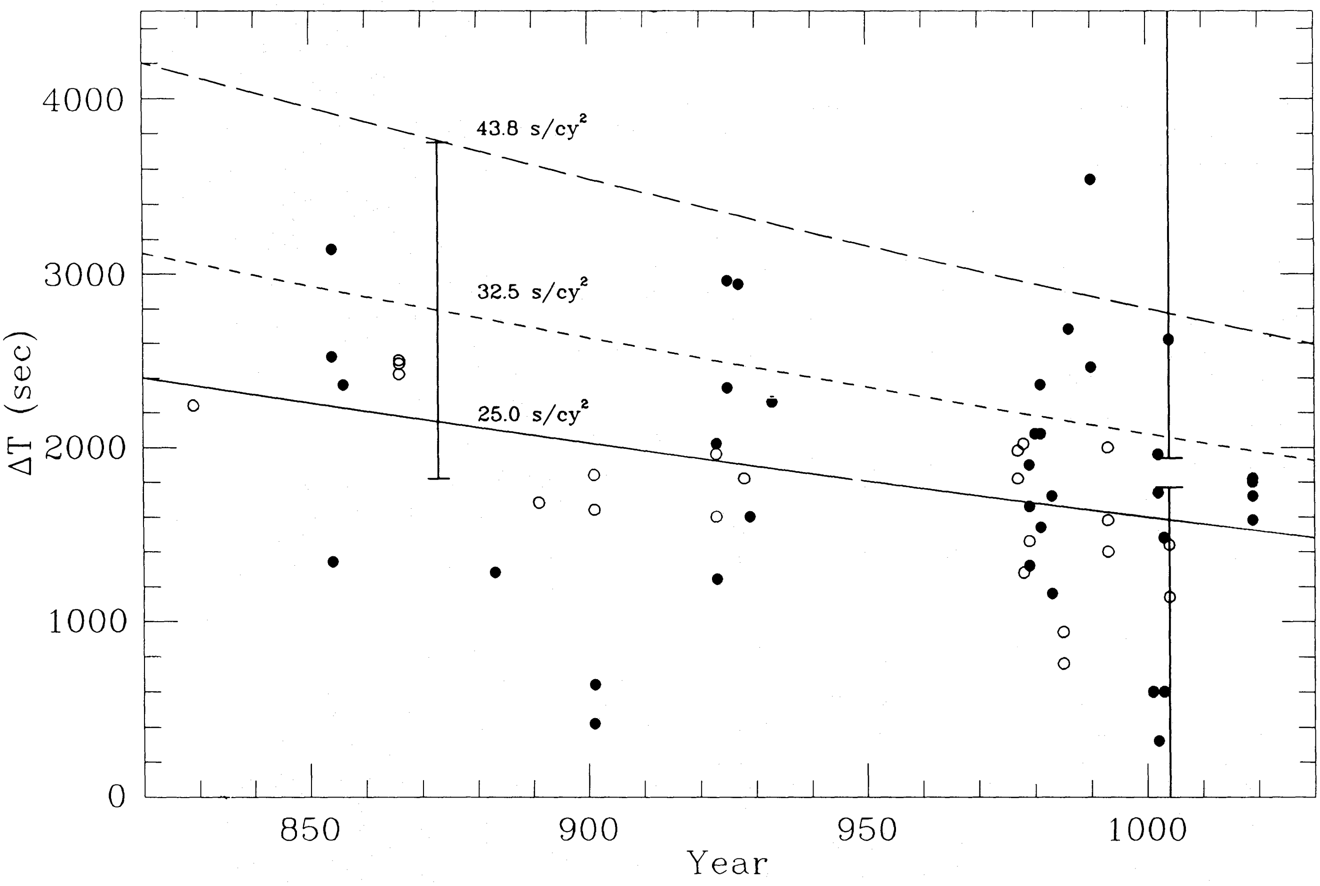

Fig. 2. ΔT values derived from medieval eclipse observations as plotted in Fig. 1. The curve shown by a full line is the best fitting parabola to the data, having equation $\Delta T = 25.0\, t^2$, where t is time in centuries measured from the reference epoch A.D. 1800. For comparison, the theoretical tidal parabola ($c = 43.8$) and the parabola derived by SM from ancient Babylonian observations ($c = 32.5$) are shown by dotted lines

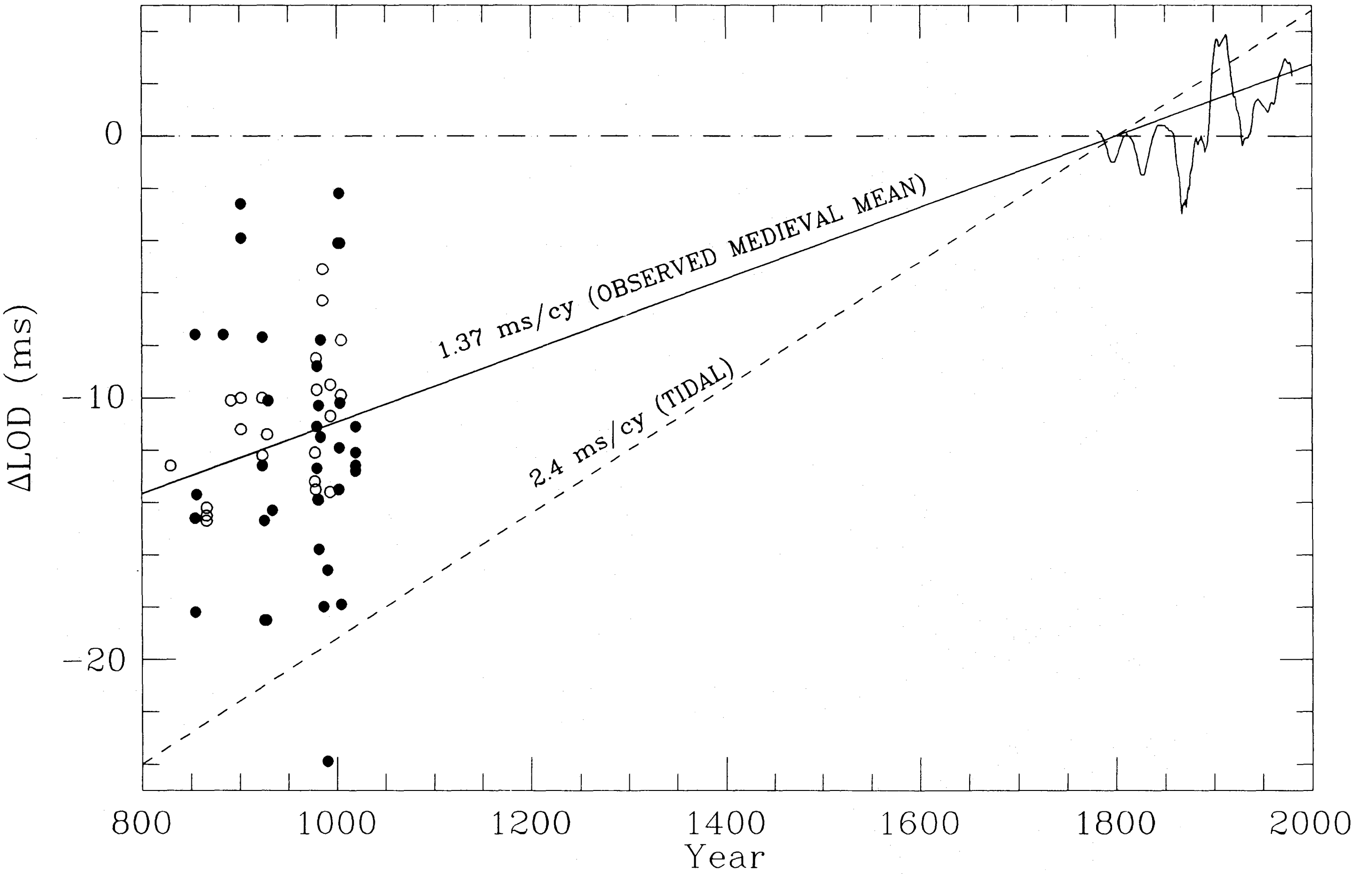

Fig. 3. Changes in the length of the day (Δ LOD) relative to the reference day of 86400 SI seconds as deduced from the medieval eclipse timings. The accurately mapped fluctuations since A.D. 1780, as derived by SM from telescopic measurements, are shown for comparison. Also shown are the best fitting straight line to the data points, which has Δ LOD = 0 at the reference epoch A.D. 1800 (full line) and the theoretical tidal variation (diagonal broken line)

fluctuations" mapped in detail by SM from telescopic observations since A.D. 1780. The quantity ΔLOD has an assumed value of zero at precisely A.D. 1800. Individual values of Δ LOD for the medieval observations are calculated from the previously obtained ΔT results using the following formula:

$$\Delta \mathrm{LOD} = 2\Delta T/36525\,(t-18)\ \mathrm{sec.}$$

The mean value of Δ LOD for the medieval data points shown is -11.6 ± 0.6 ms at an epoch very close to A.D. 950. The observed figure of -11.6 ms corresponds to an average rate of increase in the LOD over the subsequent 850 years (down to A.D. 1800) of

$$1.37 \pm 0.07\ \mathrm{ms/cy}.$$

This rate closely confirms the result of SM of 1.4 ms/cy derived from a much smaller sample of medieval timed data.

In Fig. 3 are also shown the following straight lines: (i) the observed average rate of change in the LOD of 1.37 ms/cy since medieval times (full line); (ii) the theoretical rate of change in the LOD due to lunar and solar tides of 2.4 ms/cy (broken line), which again follows from the investigation of Lambeck (1980). The latter figure would give a value of -20.4 ms for LOD at the epoch A.D. 950, a difference of 8.8 ms from the observed result of -11.6 at this time. It is anticipated that over the past few millennia, decade fluctuations of amplitude similar to those mapped in recent centuries would have continually occurred. However, it should be emphasised that the scatter in the medieval data points is to be interpreted as due to errors of measurement only.

Figure 3 clearly shows a divergence of approximately 1.0 ms/cy between the observed average rate of increase in the LOD since medieval times and the expected rate due to tides alone. During this time, the steady long-term tidal lengthening of the day has been partially offset by a substantial and probably rather variable non-tidal decrease in the LOD. Comparison with the results of SM based on ancient Babylonian observations (of mean epoch 390 B.C.) indicates that the trend determined over the past millennium differs markedly from that in the previous 1500 years. In this earlier period, the LOD had increased at a mean rate of 2.4 ms/cy, non-tidal changes averaging out virtually to zero. It seems plausible that the long-term non-tidal variations in the LOD detected using ancient and medieval eclipse observations are quasi-periodic oscillations, but the time-scale covered by even the most ancient observations may be too short to confirm this inference.

5. Conclusion

The fairly large number of medieval eclipse timings analysed here confirms the existence of significant non-tidal variations in the LOD of amplitude several ms on a time-scale of millennia. It thus seems clear that the Earth's rotational history over the last few thousand years has been rather complex. At present, the long-term resolution of changes in the LOD is still low and in our view caution should be exercised in attributing the observed variations to any single geophysical mechanism. Possible causes might be a combination of both (i) variations in the principal moment of inertia of the Earth and (ii) interchanges of angular momentum between the mantle of the Earth and the core or hydrosphere. It is perhaps worth emphasising that minor global changes in sea-level, although insignificant in so far as the constancy of tidal dissipation is concerned, could produce appreciable alterations in the Earth's moment of inertia. To produce a non-tidal decrease in the length of the day by the observed 8.8 ms between about A.D. 950 and 1800 would require a global decrease in sea-level, due to freezing of water in the polar regions, by approximately 0.5 metre in this time. Although such a result may be excessive, studies of the effects of known climatic changes on mean sea-level over this time-scale. – as well as world-wide measurements of sea-level variations – might help to give a better understanding of the contribution by other non-tidal mechanisms.

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References

al-Battānī, Abū 'Abd Allah Muḥammad ibn Sinān: c. 910, *al-Zīj al-Sabi'* (see Nallino)

al-Bīrūnī, Abū-Raiḥān Muḥammad ibn Aḥmad: 1025, *Kitab Taḥdīd Nihāyāt al-Amākin li-Taṣḥīḥ Masāfāt al-Masākin*, (ed. Muḥammad ibn Tawīt al-Ṭanjī, Ankara, 1962, pp. 271–272)

al-Bīrūnī, Abu-Raiḥān Muḥammad ibn Aḥmad: 1030, *al-Qānūn al-Mas'ūdī*, ed. M. Nizāmu' d-Dīn, Hyderabad, Deccan, 1955, II, pp. 632 and 740–742)

Ali, J.: 1967, *The Determination of the Co-ordinates of Cities* (Beirut)

Caussin, C.: 1804, *Le Livre de la grande Table Hakémite par Ebn Iounis*, (Paris, pp. 16–240)

Dickey, J.O., Williams, J.G., Newhall, X.X., Yoder, C.F.: 1984, *Proc. IAG/IUGG, Hamburg*, Ohio State Univ., Columbus, II, pp. 509–521

Freeman-Grenville, G.S.P.: 1977, *The Muslim and Christian Calendars* (Oxford)

Goldstein, B.R.: 1979, *Archives Internationale d'Histoire de Sciences* **29**, 101

IAU: 1968, *Trans. Int. Astron. Union* **13B**, 48

ibn Yūnus, Abū al-Ḥasan Alī ibn 'Abd al-Raḥmān: 1008, *al-Zīj al-Kabīr al-Ḥākimī* (see Caussin)

King, D.A.: 1976, *Dictionary of Scientific Biography* **XIV**, 574–580 (Entry: ibn Yūnus)

Lambeck, K.: 1980, *The Earth's Variable Rotation* (Cambridge, pp. 336–339)

Morrison, L.V.: 1979, *Monthly Notices Roy. Astron. Soc.* **187**, 41

Nallino, C.A.: 1899, *Al-Battānī sive Albatenii Opus Astronomicum*, (Milan)

Newcomb, S.: 1878, *Washington Observations for 1875*, App. II, 1

Newcomb, S.: 1895, *Astron. Pap. Amer. Eph.* **6**, part I

Newhall, X.X., Williams, J.G., Standish, E.M.: *Astron. Astrophys.* **125**, 150

Newton, R.R.: 1970, *Ancient Astronomical Observations and the Accelerations of the Earth and Moon* (Baltimore)

Newton, R.R.: 1972, *Memoirs Roy. Astron. Soc.* **76**, 99

Oppolzer, T. Ritter von: 1887, *Canon der Finsternisse* (Wien); (Reprinted as *Canon of Eclipses*, New York, 1962)

Smart, W.M.: 1977, *Textbook on Spherical Astronomy*, (Cambridge, p. 380)

Stephenson, F.R., Morrison, L.V.: 1984, *Phil. Trans. Roy. Soc. Lond.* **A 313**, 47