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Empirical transformations from *U.T.* to *E.T.* for the period 1800—1988

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Approximation formulae describing the long-term changes in the rotation of the Earth are evaluated. The time scale reduction from Universal to Ephemeris Time can be accomplished by a 12th degree polynomial with a mean error of less than one second for the entire time span 1800—1988. The maximum residuals of least squares fits of the 19th and 20th century data, respectively, can be limited to this accuracy by even simpler expressions.

Оцениваются приближенные формулы для долговременных изменений вращения Земли. Редукцию с всемирного на эфемеридное время можно сделать при помощи многочлена 12-й степени со средней погрешностью меньшей чем одна секунда за период 1800—1988 гг. Максимальные резидуалы решений полученных методом наименьших квадратов для данных из 19-ого и 20-ого века можно ограничить этой точностью при помощи даже более простых выражений.

Key words: ephemeris time — universal time — time scales

AAA subject category: 044

1. Introduction

The long-term changes in the rotation of the Earth directly affect the measurement of Universal Time (*U.T.*), and hence the observed positions of all bodies within the solar system will deviate by differences proportional to the mean motions of the respective bodies from those values which are based on orbit theories. These fluctuations, evaluated mainly in an extensive investigation by SPENCER JONES (1939), were used by CLEMENCE (1948) to introduce the uniform time scale Ephemeris Time (*E.T.*) into the solar system ephemerides. The tabular mean longitude of the Sun (NEWCOMB 1895) remains unchanged if *E.T.* is used as the argument of time instead of *U.T.* CLEMENCE demonstrated that the proper introduction of *E.T.* in the ephemeris of the Moon requires an additional correction term for the acceleration of the Moon's mean longitude. Accurate values for the time correction $\Delta T = U.T. - E.T.$ were derived by BROUWER (1952) who used lunar occultation timings and meridian observations. Starting in 1955, the changes in *U.T.* were obtained by means of atomic time standards. Based on BROUWER's material, SCHMADEL and ZECH (1979) (hereafter referred to as Paper I) derived numerical approximations for the quantity ΔT in order to facilitate observation time reductions of planetary bodies.

In an impressive paper STEPHENSON and MORRISON (1984) extensively rediscussed the problem of changes in the rotation of the Earth over the past 2,700 years (see their references for further papers on this topic). They reanalyzed more than 50,000 occultation timings including lunar limb corrections. STEPHENSON and MORRISON evaluated annual mean values for ΔT yielding a considerably smoother course of the data than that of BROUWER's curve under the plausible assumption of a lunar tidal acceleration of $-26''/\text{century}^2$. Since 1986 these numbers have been published as standard values in the Astronomical Almanac issues.

Using this new data body we constructed polynomial approximations for ΔT for reduction purposes of mainly minor planet observations. We therefore limited the time span covered to the period after 1800 and aspired to reach a mean error accuracy of less than one second of time which corresponds to approximately 10^{-5} days, the usually published time resolution.

2. Results

We used the annual mean values of ΔT as given in the paper of STEPHENSON and MORRISON. Until 1859 the numbers are given to the nearest tenths of a second whereas afterwards there is a $0^s\!.01$ precision. The values for 1981 to 1988 are extrapolated from the 1987 Astronomical Almanac. Fig. 1 shows the course of these data points together with the superposition of the 12th degree polynomial mentioned below.

Aside from an adequate accuracy, the interpolating function should consist of only a few coefficients. It was proved that the development into Fourier series or splines has no advantage over a simple time power series. Least squares fits revealed polynomial coefficients up to a degree of 20. The curves in Fig. 2 demonstrate the approximation quality that can be achieved with regard to the polynomial degree. The entire period considered can be covered by a 12th order equation with a mean error of less than one second. The resulting "$O - C$" difference vanishes 19 times and leaves an absolute maximum of

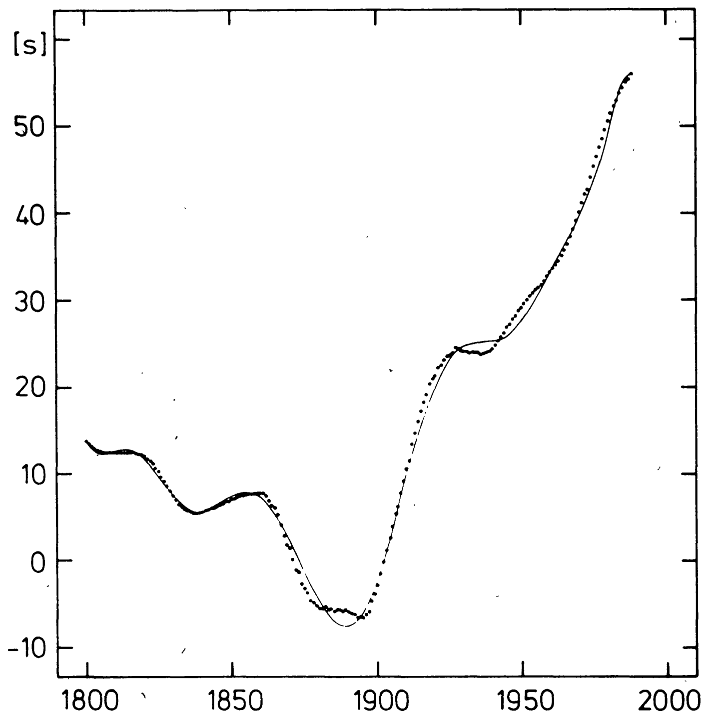

Fig. 1. Annual values for the difference ΔT 1800—1988. The solid curve represents a 12th order least-squares fit.

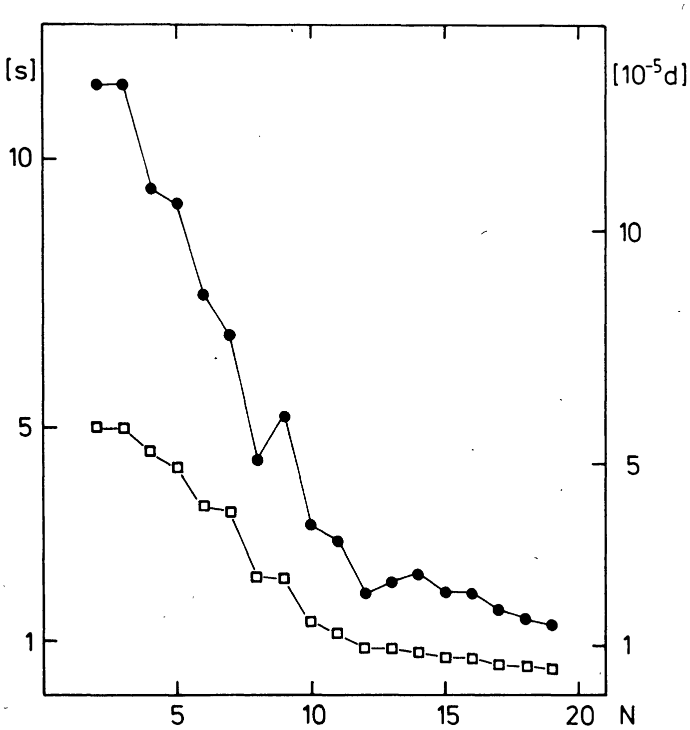

Fig. 2. Mean (open squares) and maximal errors (dots) for approximations of ΔT 1800—1988 versus polynomial degree N.

1^{s}9. For 70 per cent of all instants, the residuals do not exceed 1 s. In the following expression, which is recommended for the usual minor planet work, T denotes the time of observation elapsed since 1900 January 0^d.5 *E.T.* and is to be expressed in Julian centuries (ΔT in days):

$$\Delta T = -0.000014 + 0.001148T + 0.003357T^2 - 0.012462T^3 - 0.022542T^4 + 0.062971T^5 + 0.079441T^6 - 0.146960T^7 - 0.149279T^8 + 0.161416T^9 + 0.145932T^{10} - 0.067471T^{11} - 0.058091T^{12} . \quad (1)$$

We note that the introduced maximal errors are considerably smaller than those found in Paper I. This fact must be fully attributed to the smoothness of the Stephenson and Morrison data for the 19th century in comparison to Brouwer's values. It is impossible to cut down the maxima under the one second of time level. The use of higher order functions, however, produces the well-known detrimental effects between consecutive zeroes of the interpolation series.

For very high precision, one has to employ expressions for shorter time spans. In regard to the 19th century points, a polynomial of degree 7 is sufficient to reach the 1 s m.e. limit. The maximum errors are restricted to that level by the 10th degree approximation

$$\Delta T = -0.00009 + 0.003844T + 0.083563T^2 + 0.865736T^3 + 4.867575T^4 + 15.845535T^5 + 31.332267T^6 + 38.291999T^7 + 28.316289T^8 + 11.636204T^9 + 2.043794T^{10} . \quad (2)$$

The reduction of time scales for this century can even be accomplished (mean error $0\overset{s}{.}5$, max. error $0\overset{s}{.}95$) by means of the series of degree 7

$$\Delta T = -0.000020 + 0.000297T + 0.025184T^2 - 0.181133T^3 + 0.553040T^4 - 0.861938T^5 + 0.677066T^6 - 0.212591T^7 . \quad (3)$$

Fig. 3 gives the residual scatter of the last two equations. The use of these formulae will approximate the tabular ΔT values to within $\pm 0\overset{s}{.}5$ in about 75 per cent of all cases. It is, of course, prohibited to use any interpolation function outside of their defined validity range. As far as the first decades of the 19th century are concerned, it should be noted that the values from series (1) deviate by as much as 5 s from the approximation of Paper I. These differences arise from the STEPHENSON and MORRISON improvement in regard to BROUWER's figures.

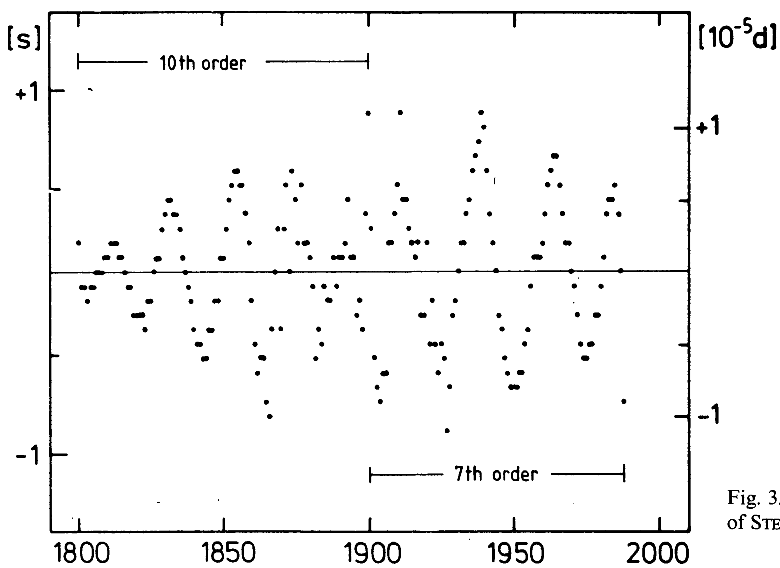

Fig. 3. Residuals of formulae (2) and (3) against the ΔT values of STEPHENSON and MORRISON.

References

BROUWER, D.: 1952, Astron. J. **57**, 125.
CLEMENCE, G. M.: 1948, Astron. J. **53**, 169.
NEWCOMB, S.: 1895, Astron. Pap. **VI**, Part I.
SCHMADEL, L. D. and ZECH, G.: 1979, Acta Astron. **29**, 101.
SPENCER JONES, H.: 1939, Mon. Not. R. Astron. Soc. **99**, 541.
STEPHENSON, F. R. and MORRISON, L. V.: 1984, Philos. Trans. R. Soc. London Ser. A **313**, 47.

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