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ELP 2000-85: a semi-analytical lunar ephemeris adequate for historical times

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Summary. New expressions for mean lunar arguments are obtained. With respect to ELP 2000-82, the main improvement consists in computing secular terms proportional to powers 3 and 4 of time. Such terms arise from secular variations of solar eccentricity and longitude of perigee, Earth figure effects and tidal effects.

A semi-analytical solution for the lunar motion, ELP 2000-85, is presented which is valid over a time span of few thousand years and easily handled. Its estimated internal precision varies from 0″.5 to about 10″ over the time span (2000 A.D.–1500 B.C.).

A comparison of ELP 2000-85 with the JPL numerical integration LE 51, over a similar time span, has been performed. The leading point of the comparison is a secular drift of 0″.0351/cy^2 in mean mean longitude between the two solutions. After removing this secular drift the differences have the same magnitude as the estimated internal precision of ELP 2000-85. For historical times, this secular drift as well as internal uncertainty of ELP 2000-85 are far below the uncertainties resulting from the tidal secular acceleration.

For practical purpose, we give in an appendix, an abridged solution usable to compute a low precision lunar ephemeris (20″) over a long time span for a given value of the tidal secular acceleration (−23″.895/cy^2).

Key words: planets and satellites: Moon – celestial mechanics

1. Introduction

The fit of ancient observations does not require a very precise lunar ephemeris but a very stable one that is to say which contains secular drifts as small as possible. Secular drift as well as other inaccuracies may result from several causes: constants used, modelisation of forces, truncation, convergency and lack of precision in the integration of the equations of motion. We shall denote as "internal precision" or "internal stability" the precision or stability resulting from the modelisation of gravitational forces and integration of the equations of motion. In this paper, we shall be mainly concerned by internal stability but we shall also discuss other factors of instability. The internal precision of the lunar ephemeris ELP 2000 (Chapront-Touzé and Chapront, 1983) is about 0″.01 over the 20th century, but it noticeably decreases over longer periods. For ancient observations, the internal precision of the Truncated Tables issued from ELP 2000 (Chapront and Chapront-Touzé, 1982) would be sufficient if it were almost constant over a long time span. The truncation level of the Tables is 0″.01, leading to maximum error of 0″.5 over one century with a total amount of about one thousand terms for the three coordinates of the Moon. Furthermore, the Truncated Tables are more easily handled because they are less bulky than complete series. Unfortunately, over a time span of several centuries, the error resulting from the internal instability of the semi-analytical solution ELP 2000-82 becomes larger than the error arising from the truncation. Hence, the leading aim of this paper is the improvement of the internal stability of the semi-analytical solution.

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2. Improvement of the internal stability of ELP 2000-82 over a long time span

In ELP 2000-82, each polar coordinate longitude V, latitude U and distance r is modeled by series:

$$w_i \delta_V + \sum_{n \geqq 0} t^n F_n(\lambda_j), \tag{1}$$

where δ_V is 1 for longitude, 0 otherwise; t is the time (TDB) measured in julian centuries from J 2000 and F_n are Fourier series of arguments $\lambda_j \cdot (\lambda_j)$ stand for planetary and lunar mean mean longitudes and Delaunay's arguments:

$$D = w_1 - T + 180°$$
$$F = w_1 - w_3$$
$$l = w_1 - w_2$$
$$l' = T - \bar{\omega}'$$

(w_i) $(i = 1, 2, 3)$, T and $\bar{\omega}'$ stand respectively for the mean mean longitude of the Moon, the mean longitude of the perigee, the mean longitude of the node, the mean mean longitude of the Earth and the mean longitude of the perihelion of the Earth. (w_i) are polynomial functions of the time:

$$w_i = \sum_{n \geqq 0} w_i^{(n)} t^n . \tag{2}$$

Polar coordinates and (w_i) are referred to the ELP reference frame: mean dynamical (inertial) ecliptic of date and departure point $\gamma'(2000)$. $\gamma'(2000)$ is referred to the mean dynamical equinox of J 2000, $\gamma(2000)$ by: $N\gamma'(2000) = N\gamma(2000)$.

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N being the ascending node of the mean dynamical ecliptic of date on the mean dynamical ecliptic of J2000.

Actually, in expressions (1) and (2), the upper limit for n is 2. This limit is reasonable for series (1), because the coefficients of trigonometric lines in F_n for $n > 2$ are very small, but is insufficient for polynomials (2) and the instability of ELP 2000-82 mainly arises from the drastic truncation of them.

In arguments (w_i), t^2 terms mainly arise from secular terms in the solar eccentricity, tidal effect, Earth figure. Further terms will arise from the same topics.

Following Brown's method (Chapront-Touzé and Chapront, 1980), the perturbations in ELP 2000-82 are derived as periodic and secular increments to 3 constants $z_j^{(0)}$ and 3 arguments w_j to be substituted in the main problem solution. We have:

$z_1^{(0)} = m$; ratio of the solarn and lunar mean motions.
$z_2^{(0)} = \Gamma$; half coefficient of $\sin F$ term in latitude.
$z_3^{(0)} = E$; half coefficient of $\sin l$ term in longitude.

For instance in the main problem the expressions for the rectangular coordinates of the Moon (x_i) and velocity (u_i) are:

$$x_i = x_i^* (z_j^{(0)}, w_j, e', \varpi', T)$$
$$u_i = u_i^* (z_j^{(0)}, w_j, e', \varpi', T) . \tag{3}$$

e' stands for mean solar eccentricity. (x_i) and (u_i) are referred to the ELP reference frame.

The main problem involves:

$$w_j = b_j (z_i^{(0)}, e') \, t + w_j^{(0)}, \quad z_j^{(0)} = \text{const}, \quad w_j^{(0)} = \text{const}.$$

If disturbing forces result from a potential $R, z_j^{(0)}$, and w_j are yielded by:

$$\left[\frac{dz_j^{(0)}}{dt}\right] = C^{-1}\left[\frac{\partial R}{\partial w_i}\right],$$
$$\left[\frac{dw_j}{dt}\right] = [b_j(z_i^{(0)}, e')] - C^{-1T}\left[\frac{\partial R}{\partial z_i^{(0)}}\right]. \tag{4}$$

In R, coordinates of the Moon must be replaced by expressions (3) before derivation. Matrix C depends on $z_j^{(0)}$ and e'. Upper symbol T stands for transposition. Consider now each individual effect leading to secular terms: contributions due to solar eccentricity and perigee, tidal and Earth figure effect.

2.1. *Secular terms in solar eccentricity and perigee*

This case is not described by Eqs. (4). Secular terms in e' and ϖ' are substituted in (3) and remaining perturbations (Chapront-Touzé, 1982) are yielded by:

$$\left[\frac{dz_j^{(0)}}{dt}\right] = C^{-1}\left[\{w_i, e'\}\frac{de'}{dt} + \{w_i, \varpi'\}\frac{d\varpi'}{dt}\right],$$
$$\left[\frac{dw_j}{dt}\right] = [b_j(z_i^{(0)}, e')] - C^{-1T}\left[\{z_i^{(0)}, e'\}\frac{de'}{dt} + \{z_i^{(0)}, \varpi'\}\frac{d\varpi'}{dt}\right]. \tag{5}$$

$\{w_i, e'\}$ and similar terms are Poisson brackets:

$$\{w_i, e'\} = \sum_j \left[\frac{\partial u_j^*}{\partial w_i}\frac{\partial x_j^*}{\partial e'} - \frac{\partial u_j^*}{\partial e'}\frac{\partial x_j^*}{\partial w_i}\right].$$

In ELP 2000-82, only t terms in e' and ϖ' were taken into account. Furthermore, Eqs. (5) were solved up to the first order, that is setting $z_j^{(0)}, e', \varpi'$ to constants and w_j to linear functions of time in C and in the Poisson brackets. The (b_j) were solely developed up to the first order in $z_k^{(0)}$ and e'. These yielded:

- periodic terms in $z_j^{(0)}$ and w_j;
- t terms in $z_j^{(0)}$ induced by e' which are very small;
- t terms in w_j induced by ϖ';
- t^2 terms in w_j induced by t terms in $z_k^{(0)}$ and e' through b_i.

The upper order terms in w_j are obtained both by taking into account upper order terms in e' and ϖ' and by solving Eqs. (5) up to further orders. But the latter requires derivatives of the main problem solution with respect to constants $z_j^{(0)}$ and e'. Actually only a solution up to the second order can be performed, first and second derivatives of ELP 2000-82 being solely available. On the other hand, secular terms in $z_j^{(0)}$ being very small, their contributions to the second order can be disregarded except for a small contribution through b_i. So the upper order terms in w_j have been derived from two sets of equations.

- First order set:

$$\left[\frac{d\Delta z_j^{(0)}}{dt}\right] = \langle C^{-1}[\{w_i, e'\}]\rangle \frac{de'}{dt}, \tag{6}$$
$$\left[\frac{d\Delta w_j}{dt}\right] = -\langle C^{-1T}[\{z_i^{(0)}, \varpi'\}]\rangle \frac{d\varpi'}{dt}, \tag{7}$$
$$\left[\frac{d\Delta w_j}{dt}\right] = \left[\sum_k \frac{\partial b_j}{\partial z_k^{(0)}}\Delta z_k^{(0)} + \frac{\partial b_j}{\partial e'}\Delta e'\right]. \tag{8}$$

- Second order set:

$$\left[\frac{d\Delta z_j^{(0)}}{dt}\right] = \frac{\partial}{\partial e'}\langle C^{-1}[\{w_i, e'\}]\rangle \frac{de'}{dt}\Delta e', \tag{9}$$
$$\left[\frac{d\Delta w_j}{dt}\right] = -\frac{\partial}{\partial e'}\langle C^{-1T}[\{z_i^{(0)}, \varpi'\}]\rangle \frac{d\varpi'}{dt}\Delta e', \tag{10}$$
$$\left[\frac{d\Delta w_j}{dt}\right] = \left[\sum_k \frac{\partial^2 b_j}{\partial z_k^{(0)}\partial e'}\Delta z_k^{(0)}\Delta e' + \frac{1}{2}\frac{\partial^2 b_j}{\partial e'^2}(\Delta e')^2\right]. \tag{11}$$

$\langle F\rangle$ stands for the constant term of the Fourier series $F(\lambda_j)$ which is independent of each λ_k and $\Delta e'$ for $(e' - e'_0)$. Numerical values of the first and second derivatives of b_i are given in Table 1.

Table 1. First and second derivatives of lunar motions with respect to constants $z_j^{(0)}$ and e' · b_i are measured in arcsec/cy

Derivatives	b_1	b_2	b_3
$\partial/\partial m$	−23162146310	343212639	−86764528
$\partial/\partial\Gamma$	0	−7766021	1157847
$\partial/\partial E$	0	−1910121	−2248987
$\partial/\partial e'$	0	1829690	−308445
$10^{-4}\,\partial^2/\partial m^2$	61929772	577404	23538
$10^{-4}\,\partial^2/\partial m\,\partial\Gamma$	0	−15900	1323
$10^{-4}\,\partial^2/\partial m\,\partial E$	0	−6109	−4201
$10^{-2}\,\partial^2/\partial m\,\partial e'$	0	659199	−27039
$10^{-4}\,\partial^2/\partial\Gamma^2$	0	−17416	2589
$10^{-4}\,\partial^2/\partial\Gamma\,\partial E$	0	−60	−2
$10^{-2}\,\partial^2/\partial\Gamma\,\partial e'$	0	−8409	460
$10^{-4}\,\partial^2/\partial E^2$	0	−3489	−4108
$10^{-2}\,\partial^2/\partial E\,\partial e'$	0	−4165	−2492
$\partial^2/\partial e'^2$	0	109967808	−18484652

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Table 2. Solar secular terms from Laskar's results. $\bar\omega'$ is measured in arcsec, t is measured in julian centuries from J 2000

e'_1	$-0.420373315\,10^{-4}$
e'_2	$-0.126775602\,10^{-6}$
e'_3	$0.144316528\,10^{-9}$
$\bar\omega'_1$	1161.22834
$\bar\omega'_2$	0.532691947
$\bar\omega'_3$	−0.000138457

The secular terms in e' and $\bar\omega'$ up to degree 3, issued from Laskar's results (Laskar, 1986), have been taken into account. They are quoted in Table 2:

$$e' = e'_0 + \sum_{k=1}^{3} e'_k t^k$$

$$\bar\omega' = \bar\omega'_0 + \sum_{k=1}^{3} \bar\omega'_k t^k .$$

They induce secular terms in w_i up to degree 4. The separate contributions and total results are given in Table 3. The t^2 terms are slightly different from those given in (Chapront-Touzé and Chapront, 1983) because they have been computed with a greater accuracy. The t^3 terms are complete but the t^4 are not because third derivatives of the main problem are missing. Nevertheless, the contributions of those derivatives can be expected to be small while considering literal developments of the main problem solution. Evaluate an order of magnitude of these t^4 terms which have been neglected. Contribution through b_j would be:

$$\frac{d\Delta w_j}{dt} = \frac{1}{6}\frac{\partial^3 b_j}{\partial e'^3}(e'_1 t)^3 . \qquad (12)$$

Since b_j contains only even powers of e' (Brouwer and Clemence, 1961):

$$\frac{\partial^3 b_j}{\partial e'^3} \approx e'_0 \frac{\partial^2 b_j}{\partial e'^2} \times \text{const} .$$

Hence, in (12) the coefficient of t^3 can be evaluated as:

$$e'^3_1 \frac{\partial^3 b_j}{\partial e'^3} \approx \frac{e'^3_1 e'_0}{e'_1 e'_2}\left(\frac{\partial^2 b_j}{\partial e'^2} e'_1 e'_2\right) \times \text{const}$$

involving the quantity $((\partial^2 b_j)/(\partial e'^2)(e'_1 e'_2))$ which is already known when computing the t^2 terms in (11). Finally the secular terms from (12) would be approximately less than those from (11) by a factor of 10^3.

The remaining contributions would be:

$$\left[\frac{dz_j^{(0)}}{dt}\right] = \frac{1}{2}\frac{\partial^2}{\partial e'^2}\langle C^{-1}[\{w_i, e'\}]\rangle (e'_1 t)^3 . \qquad (13)$$

x_i^* and u_i^* can be modeled as Fourier series in $D, l', l, F, \bar\omega'$ whose coefficients depend on $z_j^{(0)}$ and e':

$$\sum A_{i_1 \ldots i_5}\begin{bmatrix}\sin\\ \cos\end{bmatrix}(i_1 D + i_2 l' + i_3 l + i_4 F + i_5 \bar\omega')$$

Table 3. Contributions of secular terms in solar eccentricity to mean lunar arguments. The first column refers to the contribution involved in each lunar secular term. The second column refers to the eqs numbers in the present paper, from 6 to 11, where this contribution appears. w_i are measured in arcsec, t in julian centuries from J2000

	Eqs.	m	Γ	E	w_1	w_2	w_3
t terms							
$e'_1, \bar\omega'_1$	6, 7	$-0.506343\ 10^{-9}$	$0.176409\ 10^{-9}$	$0.235922\ 10^{-8}$	−0.00567	0.03388	0.00003
t^2 terms							
$e'_2, \bar\omega'_2$	6, 7	$-0.152765\ 10^{-11}$	$0.528637\ 10^{-12}$	$0.708918\ 10^{-11}$	0	0.00002	0
$e'_1, \bar\omega'_1$	9, 10	$0.619006\ 10^{-12}$	$-0.221087\ 10^{-12}$	$-0.296276\ 10^{-11}$	0.00001	−0.00009	0
$\Delta m, \Delta\Gamma, \Delta E$	8				5.86399	−0.08983	0.01942
e'_1	8				0	−38.45764	6.48310
Total		$-0.908644\ 10^{-12}$	$0.307550\ 10^{-12}$	$0.412642\ 10^{-11}$	5.86400	−38.54754	6.50252
t^3 terms							
$e'_3, \bar\omega'_3$	6, 7	$0.173902\ 10^{-14}$	$-0.601780\ 10^{-15}$	$-0.807005\ 10^{-14}$	0	0	0
e'_1, e'_2 $\bar\omega'_1, \bar\omega'_2$	9, 10	$0.373358\ 10^{-14}$	$-0.133350\ 10^{-14}$	$-0.178701\ 10^{-13}$	0	−0.0000002	0
$\Delta m, \Delta\Gamma, \Delta E$	8				0.0070154	−0.0001074	0.0000233
e'_2	8				0	−0.0773200	0.0130344
$\Delta m, \Delta\Gamma,$ $\Delta E, e'_1$	11				0	0.0323885	−0.0054442
Total		$0.547260\ 10^{-14}$	$-0.193528\ 10^{-14}$	$-0.259401\ 10^{-13}$	0.0070154	−0.0450391	0.0076135
t^4 terms							
$\Delta m, \Delta\Gamma, \Delta E$	8				−0.000031689	0.000000486	−0.000000105
$\Delta m, \Delta\Gamma, \Delta E,$ e'_1, e'_2	11				0	0.000146515	−0.000024628
e'_3	8				0	0.000066014	−0.000011128
Total					−0.000031689	0.000213015	−0.000035861

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and,

$$A_{i_1 \ldots i_5} = e'^{i_2} P(e'^2, z_j^{(0)}).$$

P stands for polynomial series.

So $\frac{\partial x_i^*}{\partial e'}$ and $\frac{\partial y_i^*}{\partial e'}$ will be of a similar form with:

$$A_{i_1 \ldots i_5} = e'^{i_2 - 1} P(e'^2, z_j^{(0)}) \qquad i_2 \neq 0$$
$$A_{i_1 \ldots i_5} = e' P(e'^2, z_j^{(0)}) \qquad i_2 = 0.$$

It can be deduced from expression of C (Chapront-Touzé and Chapront, 1980) that it contains only even powers of e'.

Then $\langle C^{-1}[\{w_i, e'\}]\rangle$ will be of the form: $e'[P(e'^2, z_j^{(0)})]$.

Hence, the coefficient of the t^3 term in (13) can be evaluated as:

$$\frac{1}{2}\frac{\partial^2}{\partial e'^2}\langle C^{-1}[\{w_i, e'\}]\rangle\, e'^3_1$$

$$\approx \frac{e'^2_1 e'_0}{e'_2}\frac{\partial}{\partial e'}\langle C^{-1}[\{w_i, e'\}]\rangle (3e'_1 e'_2) \times \text{const}.$$

Finally the secular terms from (13) would be also approximately less than those from (9) by a factor of 10^3.

2.2. *Tidal effects*

In ELP 2000-82, following (Williams et al., 1978), tidal effects are modeled by forces whose equatorial components (X_i^E) are:

$$[X_i^E] = -3k_2 \frac{Gm_L}{r^3}\left(1 + \frac{m_L}{m_T}\right)\frac{a_T^5}{r^5} \begin{bmatrix} x + y\delta \\ y - x\delta \\ z \end{bmatrix}$$

In the above equations x, y, z stand for true equatorial coordinates of the Moon, k_2 for Love number, δ for phase and r for the Earth-Moon distance. These forces do not result from a potential. Following (Lestrade and Chapront-Touzé, 1982), Eqs. (4) are replaced by:

$$\left[\frac{dz_j^{(0)}}{dt}\right] = \sum_i C^{-1} X_i \left[\frac{\partial x_i^*}{\partial w_k}\right]$$
$$\left[\frac{dw_j}{dt}\right] = [b_j(z_i^{(0)}, e')] - \sum_i C^{-1T} X_i \left[\frac{\partial x_i^*}{\partial z_k^{(0)}}\right]. \qquad (14)$$

X_i are components of the tidal forces in the ELP reference frame. Rotation of the true equatorial reference frame of date into the ELP 2000 reference frame involves ε (the inclination of the true equator of date with respect to the mean ecliptic of date) and precession ψ; x_i^* involves $\bar{\omega}'$.

In ELP 2000-82, ψ has been set to zero, ε and $\bar{\omega}'$ to constants, and Eqs. (14) have been solved up to the first order.

The t terms in ψ, ε and $\bar{\omega}'$ yield t^2 terms in $z_j^{(0)}$ and t^3 terms in w_i through the development up to the first order. Resolution of Eqs. (14) up to the second order also induces secular terms in w_j solely derived from secular terms of solar eccentricity through:

$$\left[\frac{d\Delta z_j^{(0)}}{dt}\right] = \frac{\partial}{\partial e'}\left\langle \sum_i C^{-1} X_i \left[\frac{\partial x_i^*}{\partial w_k}\right]\right\rangle \Delta e'$$

$$\frac{d\Delta w_j}{dt} = \sum_k \frac{\partial b_j}{\partial z_k^{(0)}} \Delta z_k^{(0)}.$$

Here, only the t terms of e' are taken into account. The t^4 terms induced by tidal effect should be negligible. Separate contributions and total results are gathered in Table 4. All of them are proportional to $k_2\delta$. The numerical value of $k_2\delta$ used in the JPL numerical integration DE 200 (Standish, 1981) and in published ephemerides is:

$$k_2\delta = 0.01221 \qquad (15)$$

2.3. *Earth figure*

The potential depends on ε and ψ. Equations (4) involve $\bar{\omega}'$ through x_i^*.

In ELP 2000-82, only t terms in ε, $\bar{\omega}'$ and ψ have been taken into account. The t^2 terms in ε, $\bar{\omega}'$ and ψ yield secular terms quoted in Table 5. The t^3 terms and integration of Eqs. (4) up to the second order should yield negligible secular terms in w_i.

2.4. *New expressions for mean lunar arguments*

We give in Table 6 expressions for the mean lunar and solar arguments and Delaunay arguments related to the ELP 2000 constants. The latter are fitted to the JPL numerical integration DE 200 by comparing the semi-analytical solution ELP 2000-82 to DE 200 all over the 20th century. (w_i), as defined in Sect. 2, are referred to departure point. L, $\bar{\omega}$, and Ω are similar quantities

Table 4. Tidal contributions to mean lunar arguments. The first column refers to the contributions involved in each lunar secular term. w_i are measured in arcsec, t in julian centuries from J2000

	$m/k_2\delta$	$\Gamma/k_2\delta$	$E/k_2\delta$	$w_1/k_2\delta$	$w_2/k_2\delta$	$w_3/k_2\delta$
t terms						
	0.844901 10^{-7}	−0.895810 10^{-8}	0.114569 10^{-6}			
t^2 terms						
$\varepsilon_1, \bar{\omega}'_1, \psi_1$	0.415698 10^{-11}	−0.440746 10^{-12}	0.563689 10^{-11}			
e'_1	−0.807673 10^{-13}	−0.584105 10^{-13}	0.927026 10^{-13}			
$\Delta m, \Delta\Gamma, \Delta E$				−978.486	14.4244	−3.79939
t^3 terms						
$\Delta m, \Delta\Gamma, \Delta E$				−0.0320948	0.0004731	−0.0001246
$\Delta m, \Delta\Gamma, \Delta E$				0.0006236	−0.0000091	0.0000022
Total				−0.0314712	0.0004640	−0.0001224

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Table 5. Contribution of Earth figure to mean lunar arguments. The first column refers to the contributions involved in each lunar secular term. w_i are measured in arcsec, t in julian centuries from J2000

	m	Γ	E	w_1	w_2	w_3
t^2 *terms*						
ε_1				0.1925	0.1003	−0.0958
t^3 *terms*						
$\varepsilon_2, \bar{\omega}'_2, \psi_2$	0	0	0	−0.0000268	−0.0000139	0.0000133

Table 6. New arguments for the ephemeris ELP 2000. t (TDB) is measured in julian centuries from J 2000 (JD 2451545.0). Arguments are measured in arcseconds except for constant terms

w_1	$218°18'59''.95571 + 1732559343.73604\,t - 5.8883\,t^2 + 0.006604\,t^3 - 0.00003169\,t^4$
w_2	$83°21'11''.67475 + 14643420.2632\,t - 38.2776\,t^2 - 0.045047\,t^3 + 0.00021301\,t^4$
w_3	$125°02'40''.39816 - 6967919.3622\,t + 6.3622\,t^2 + 0.007625\,t^3 - 0.00003586\,t^4$
L	$218°18'59''.95571 + 1732564372.83264\,t - 4.7763\,t^2 + 0.006681\,t^3 - 0.00005522\,t^4$
$\bar{\omega}$	$83°21'11''.67475 + 14648449.3598\,t - 37.1656\,t^2 - 0.044970\,t^3 + 0.00018948\,t^4$
Ω	$125°02'40''.39816 - 6962890.2656\,t + 7.4742\,t^2 + 0.007702\,t^3 - 0.00005939\,t^4$
D	$297°51'00''.73512 + 1602961601.4603\,t - 5.8681\,t^2 + 0.006595\,t^3 - 0.00003184\,t^4$
l'	$357°31'44''.79306 + 129596581.0474\,t - 0.5529\,t^2 + 0.000147\,t^3$
l	$134°57'48''.28096 + 1717915923.4728\,t + 32.3893\,t^2 + 0.051651\,t^3 - 0.00024470\,t^4$
F	$93°16'19''.55755 + 1739527263.0983\,t - 12.2505\,t^2 - 0.001021\,t^3 + 0.00000417\,t^4$
T	$100°27'59''.22059 + 129597742.2758\,t - 0.0202\,t^2 + 0.000009\,t^3 + 0.00000015\,t^4$
$\bar{\omega}'$	$102°56'14''.42753 + 1161.2283\,t + 0.5327\,t^2 - 0.000138\,t^3$

referred to the mean dynamical (inertial) equinox of date. T and $\bar{\omega}'$ are referred to the mean dynamical equinox of J2000. These expressions must be substituted in ELP 2000-82 series instead of those given in Table 11 in Chapront-Touzé and Chapront (1983) in order to improve the stability of the lunar ephemeris ELP 2000. For lunar arguments terms of degree 3 and 4 result from this paper. The t^2 terms are slightly different from those given in the mentioned Table 11 because of two small corrections: the contribution from lunar figure, which was certainly due to a lack of precision (Chapront-Touzé, 1983), has been removed and the contribution from solar eccentricity has been computed with a higher precision. Upper order secular terms in l', $\bar{\omega}'$, and T and precession are derived from Laskar's results (Laskar, 1986).

We gathered in Table 7 secular terms as they appear in this paper, in the Improved Lunar Ephemeris and in the Supplement to the Astronomical Almanac for 1984, p. S26. The latter are derived from a fit of Brown-Eckert's solution to the JPL numerical integration LE 51 (Van Flandern, 1981). Discrepancies between the t^2 terms mainly arise from the constant involved for tidal effect. The t^3 terms of this paper are quite close to Brown-Eckert's values (ILE) except for l'. Discrepancies with Van Flandern's values (Astronomical Almanac) for t^3 terms in D, l, F arise from t^3 term in L which solely has been modified in Van Flandern's work. His t^3 terms in Ω and $\bar{\omega}$ are the same as Brown-Eckert's. This discrepancy is probably due to inaccuracies in the periodic terms in Brown-Eckert's solution. Discrepancies between t^3 terms in l' arise from discrepancies between Laskar's secular terms and Newcomb's ones. The t^4 terms exist neither in Brown-Eckert's expressions nor in Van Flandern's ones.

3. ELP 2000-85

ELP 2000-85 provides semi-analytical series for ΔV, latitude U, Earth-Moon distance r. Longitude V or V_M can be derived from ΔV by:

$$V = w_1 + \Delta V \tag{16}$$

or by

$$V_M = L + \Delta V. \tag{17}$$

Equation (16) yields lunar coordinates referred to the ELP reference frame. Equation (17) yields lunar coordinates referred to mean dynamical (inertial) ecliptic and equinox of date. Transformation rotating the ELP reference frame into the mean dynamical ecliptic and equinox of J2000 is given in the next section.

Since ancient observations do not require a very accurate lunar ephemeris, the new solution ELP 2000-85 has been built from ELP 2000-82 by keeping coefficients greater than 0''.01 in Fourier series for ΔV and U and 20 m for r, coefficients greater than 0''.0003 (60 cm for r) in t Poisson series, 0''.00001 (2 cm for r) in t^2 Poisson series. These coefficients have been modified by involving lunar constants fitted to DE 200.

Expressions of Table 6 must be substituted for Delaunay arguments in main problem series and for w_1 or L in (16) or (17). In perturbations series, lunar and solar arguments are limited to their linear parts: $\bar{D}$, $\bar{l}'$, $\bar{l}$, $\bar{F}$, and $\bar{T}$. Planetary arguments are the same as in (Chapront-Touzé and Chapront, 1983). Notice that expressions

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Table 7. Comparison between 3 sets of mean lunar and solar arguments. Time is measured in julian centuries from J 2000. Arguments are measured in arcsec

Arguments	This paper	ILE	Astronomical Almanac
t terms			
D	1602961601.4603	1602961600.8604	1602961601.328
l'	129596581.0474	129596577.9840	129596581.224
l	1717915923.4728	1717915923.1254	1717915922.633
F	1739527263.0983	1739527267.4164	1739527263.137
Ω	− 6962890.2656	− 6962896.2460	− 6962890.539
t^2 *terms*			
D	− 5.8681	− 5.1496	− 6.891
l'	− 0.5529	− 0.5760	− 0.577
l	32.3893	33.2454	31.310
F	−12.2505	−11.5636	−13.257
Ω	7.4742	7.5040	7.455
t^3 *terms*			
D	0.006595	0.00680	0.019
l'	0.000147	−0.01200	−0.012
l	0.051651	0.05180	0.064
F	−0.001021	−0.00120	0.011
Ω	0.007702	0.00799	0.008

Table 8. Lunar arguments for comparision of ELP 2000-85 to LE 51. t is measured in julian centuries from J2000. Arguments are measured in arcsec except for constant terms

w_1	$218°19'00''.19748 + 1732559343.19572\,t - 7.0933\,t^2 + 0.006565\,t^3 - 0.00003169\,t^4$
D	$297°51'00''.66219 + 1602961600.8820\,t - 7.0731\,t^2 + 0.006556\,t^3 - 0.00003184\,t^4$
l'	$357°31'44''.83151 + 129596581.0996\,t - 0.5529\,t^2 + 0.000147\,t^3$
l	$134°57'48''.18396 + 1717915922.8022\,t + 31.1665\,t^2 + 0.051612\,t^3 - 0.00024470\,t^4$
F	$93°16'19''.50159 + 1739527262.7141\,t - 13.4508\,t^2 - 0.001060\,t^3 + 0.00000417\,t^4$

of Table 6 can be modified by using expressions of Table 4, if the adopted value of $k_2\delta$ is different from (15).

The time argument can be regarded as "dynamical time". An expression for converting universal time into dynamical time, for historical periods, is given by (Morrison and Stephenson, 1982).

Following (Chapront and Chapront-Touzé, 1982), the internal precision of ELP 2000-85 over the 20th century is about 0''.5 for longitude and latitude and 500 m for distance. In 1500 B.C., the internal precision mainly depends on neglected secular terms. Since the t^3 term in L amounts to 280'' and t^4 term to 48'', the expected internal precision for ELP 2000-85 at that time should be about 10''. In order to check these estimates, a comparison of ELP 2000-85 to JPL numerical integration LE 51 (Newhall et al., 1983) has been performed.

4. Comparison of ELP 2000-85 to LE 51

For this comparison, developments of lunar arguments related to constants fitted to LE 51 (Chapront-Touzé and Chapront, 1983) have been introduced instead of those of Table 6. They are quoted in Table 8. The constants have been obtained by comparing the semi-analytical solution ELP 2000-82 to LE 51 over the 20th century only. The contributions involved are exactly the same as described in Sect. 2.4.

Constant for tidal effect is:

$$k_2\delta = 0.0134415\,.$$

Several transformations have been performed in order to have both ephemerides referred to the same reference frame.

– The reference frame of LE 51 is a mean equator and equinox of B1950. The latter has been rotated into a J2000 equator and equinox denoted as DE 102 reference frame 2000, by means of precession from (Lieske et al., 1977).

– Rectangular coordinates (x_i) in the ELP reference frame have been computed from ELP 2000-85 series and (16) by:

$$x_1 = r\cos V\cos U$$
$$x_2 = r\sin V\cos U$$
$$x_3 = r\sin U\,.$$

ELP 2000 reference frame has been rotated into the mean dynamical ecliptic and equinox of J2000 by means of transformation:

$$[x_i'] = \begin{bmatrix} 1-2p^2 & 2pq & 2p\sqrt{1-\gamma^2} \\ 2pq & 1-2q^2 & -2q\sqrt{1-\gamma^2} \\ -2p\sqrt{1-\gamma^2} & 2q\sqrt{1-\gamma^2} & 1-2\gamma^2 \end{bmatrix} [x_i]\,.$$

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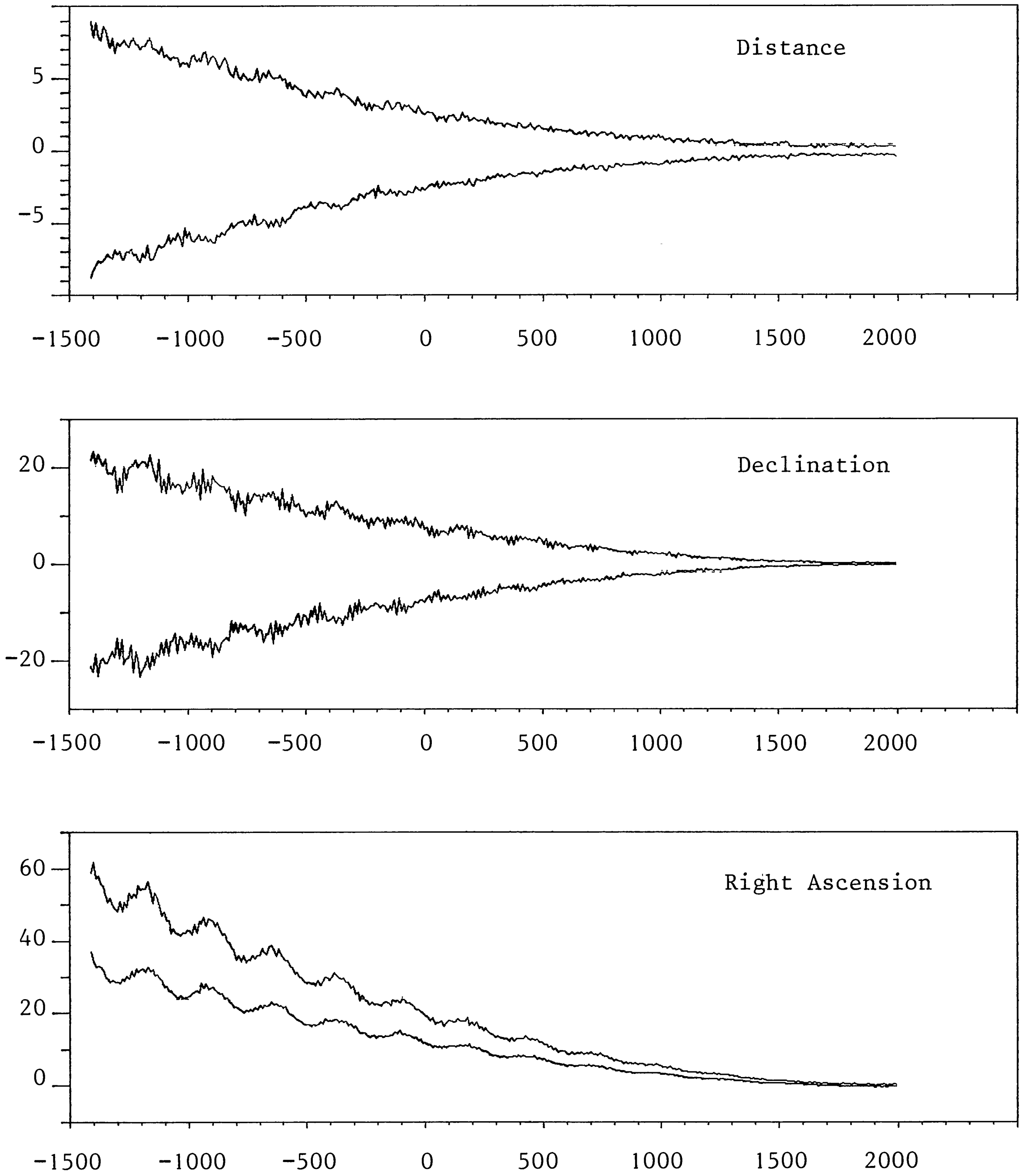

Fig. 1. Minimum and maximum of the differences LE 51 – ELP 2000-85 over 3600 days, with a step of 30 days, for the time span (1500 B.C. – 2000 A.D.). Right ascension and declination are measured in arcseconds, distance in kilometers

p and q are Laskar's series (Laskar, 1986) reproduced here up to degree 5:

$$p = 0.0010180391\,\tau + 0.0047020439\,\tau^2 - 0.0005417367\,\tau^3 - 0.0002507948\,\tau^4 + 0.0000463486\,\tau^5$$

$$q = -0.0113469002\,\tau + 0.0012372674\,\tau^2 + 0.0012654170\,\tau^3 - 0.0001371808\,\tau^4 - 0.0000320334\,\tau^5 .$$

τ stands for the time measured in unit of 100 julian centuries from J2000 and γ for $(p^2+q^2)^{1/2}$.

– The mean dynamical ecliptic and equinox of J2000 has been rotated into the DE 102 reference frame 2000 by means of:

$$[x_i''] = \begin{bmatrix} 1 & -\Delta\phi\cos\varepsilon & \Delta\phi\sin\varepsilon \\ \Delta\phi & \cos\varepsilon & -\sin\varepsilon \\ 0 & \sin\varepsilon & \cos\varepsilon \end{bmatrix} [x_i'] .$$

(x_i'') stand for lunar rectangular coordinates in the DE 102 reference frame 2000;

$\varepsilon = \varepsilon_0 + \Delta\varepsilon$.

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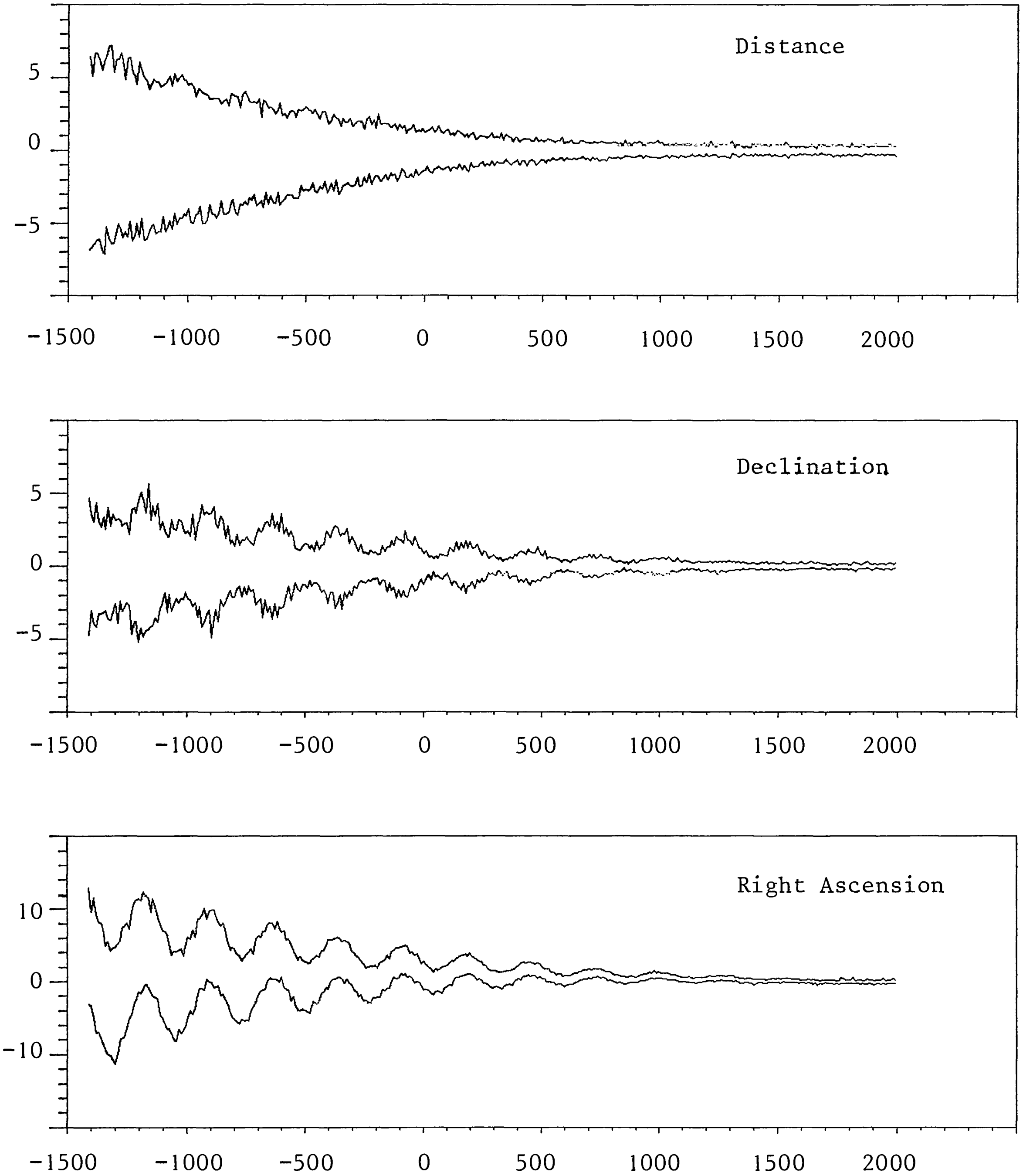

Fig. 2. Minimum and maximum of the corrected differences LE 51 − ELP 2000-85 over 3600 days with a step of 30 days for the time span (1500 B.C. − 2000 A.D.). Correction consists in adding a secular term of 0.0351 t^2 arcsec to the mean mean longitude of the Moon in ELP 2000-85. Right ascension and declination are measured in arcsec, distance in kilometers

ε_0 is the IAU value for the obliquity:

$\varepsilon_0 = 23°26'21''.448$.

$\Delta\varepsilon$ and $\Delta\phi$ are quoted in (Chapront and Chapront-Touzé, 1981):

$\Delta\varepsilon = -0''.039$,
$\Delta\phi = -0''.230$.

Differences of LE 51 and ELP 2000-85 are drawn on Fig. 1. Right ascension exhibits a secular drift. Several attempts have been made in order to fit various polynomials. It appeared that a quadratic monomial represents the best fit. The drift can be removed by adding an empirical term in w_1 to ELP 2000-85:

$\Delta w_1 = 0.0351\, t^2$ arcsec

or by substracting it from LE 51. The results are drawn on Fig. 2. In 1400 B.C., Δw_1 makes the differences in declination smaller than 5″ and the differences in right ascension smaller than 13″ which agree with the expected accuracy of ELP 2000-85 and proves the validity of the t^3 and t^4 terms derived from this paper. A residual oscillation with a period of about 270 yr clearly appears

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in Fig. 2. This period is close to the one of the argument $18 V_e - 16 T - l$, where V_e is the mean mean longitude of Venus. This oscillation is probably due to missing planetary perturbations of the form: $t^2 \sin(18 V_e - 16 T - l + \text{phase})$ which have not been yet computed.

Until now, no explanation has been found to the secular drift Δw_1. Force models, especially for tidal forces, are apparently the same in the both ephemerides and it seems very unlikely that such a large error should exist in the t^2 terms of ELP 2000-85. Δw_1 can be compared to values of bias parameter in $w_1^{(2)}$ derived from (Chapront-Touzé and Chapront, 1983) though the weak precision of ELP 2000-85 does not allow to be sure that the drift Δw_1 is still significant at an epoch near 2000, and though the missing t^3 term may have spoiled the determination of the bias parameters in the comparison of ELP 2000-82 to numerical integrations. Taking into account the corrections on the t^2 terms in w_1 quoted in Sect. 2.4, the bias parameter resulting from the comparison of ELP 2000-82 to LE 51 over the 20th century is changed to $0''.0088/\text{cy}^2$. It is smaller than Δw_1 by a factor of 4. The bias parameter resulting from the comparison of ELP 2000-82 to DE 200 over the 20th century is changed to $-0''.0193/\text{cy}^2$.

On the other hand, the authors of LE 51 provide an estimation for the internal secular drift of it (Newhall et al., 1983). With the notations of this paper, their expression changes to:

$$(\Delta w_1)_{\text{LE }51} = -0.031\,(t + 0.305)^{1.7}\ \text{arcsec}.$$

We note that Δw_1 and $(\Delta w_1)_{\text{LE }51}$ have opposite signs. The effect from $(\Delta w_1)_{\text{LE }51}$ is larger than the one from Δw_1 for epoch near 2000 and smaller for historical times, by a factor of 3.3 in 1400 B.C. But $(\Delta w_1)_{\text{LE }51}$ derives from an estimate over a time span of 20 yr included in the 20th century and may be not comparable with Δw_1.

5. Precision of ELP 2000-85

As mentioned in Sect. 4, the internal precision of ELP 2000-85 varies from $0''.5$ to $13''$ on the time span [2000 A.D.; 1500 B.C.]. The global precision of the ephemeris depends also on the precision of the constants involved and on the quality of the model for non gravitational forces. Here, non gravitational forces are confined to tidal forces only, and the model used is described in Sect. 2.2).

Because of the truncation level of ELP 2000-85 series, this model yields only t^2 and t^3 terms in the lunar arguments. The latter depends on the constant $k_2\delta$. But actually, the constant "fitted to observations" is the tidal t^2 term in the lunar mean longitude, $k_2\delta$ being deduced from it and from the model. Finally, the model for tidal forces contributes only to small t^2 terms in the mean longitudes of perigee and node, and to tidal t^3 terms which are smaller than the t^3 terms induced by solar eccentricity by a factor of 20.

ELP 2000-85 involves IAU values for masses, and we may assume that they are accurate enough for our purpose. For lunar and solar constants, ELP 2000-85 involves DE 200 values or values fitted to DE 200 (orbital parameters). The latter have been introduced because DE 200 stands for a reference ephemeris, but other sets of constants exist, for example, orbital parameters fitted to DE 102 as used in Sect. 4, or the ones obtained by fitting ELP 2000-82 to lunar occultations by Sôma (1985). All of them induce effects of $0''.5$ at most on the lunar mean longitude, except for the lunar mean motion ν and the tidal t^2 term of the mean longitude which induce secular drifts. In 1500 B.C. the discrepancy in the mean lunar longitude yielded by DE 102 value of ν is $+18''.9$ and the one yielded by Sôma's value is $-14''.5$.

Recent determinations of the tidal t^2 term in the lunar mean longitude are given in Dickey et al. (1982) and Krasinsky et al. (1985). Dickey's value ($-11''.9/\text{cy}^2$) is very near of the one used in ELP 2000-85 (-11.9473) but the stated uncertainty amounts to $\pm 0''.75/\text{cy}^2$ which could yield a discrepancy in the mean lunar longitude as large as $\pm 919''$ in 1500 B.C. Krasinsky gives two values (-11.45 and -11.1), the second one being obtained by comparison of ELP 2000 to observations. Krasinsky's determinations yield discrepancies of $+609''$ and $+1038''$ respectively in 1500 B.C. Hence, in spite of recent improvements, tidal acceleration is the main factor of inaccuracy in the lunar ephemeris for historical times.

6. Conclusion

This paper provides lunar mean arguments up to degree 4. They have been obtained in a mere analytical manner but they are related to the JPL numerical integration DE 2000 through the involved constants.

From these lunar mean arguments and from the lunar ephemeris ELP 2000 we derive an abridged solution ELP 2000-85 which can be easily used for comparison with historical astronomical observations. Its estimated internal uncertainty in longitude varies from $0''.5$ to $13''$ on the time span [2000 A.D. – 1500 B.C.] and increases before 1500 B.C. A diskette providing the complete solution ELP 2000-85 can be asked to the authors for scientific use only.

ELP 2000 has been compared to the JPL numerical integration LE 51 on the time span [2000 A.D. – 1400 B.C.]. The leading discrepancy is a secular drift in longitude. For historical times this secular drift as well as internal uncertainty of ELP 2000-85 are far below the uncertainties resulting from the tidal secular acceleration.

Appendix

Table 9 provides the leading terms of ELP 2000-85 following the general formulation:

$$\sum A(Z_1, n, Z_2)\, t^n \sin(i_{Ve} V_e + i_T T + i_{Ma} M_a + i_{Ju} J_u + i_L L + i_D D + i_{l'} l' + i_l l + i_F F + \varphi).$$

Z_1 stands for V, U, or r, Z_2 stands for C (main problem) or P (perturbations). V_e, M_a, and J_u stand for mean mean longitudes of Venus, Mars and Jupiter. φ is a phase which amounts to 0 for main problem series and t^2 perturbations in ΔV and U, and 90° for corresponding series in r. In the other cases, the values of φ are given in Table 9.

In main problem series (C) arguments D, l', l, F of Table 6 shall be used. In perturbation series (P) these arguments are limited to their linear parts: $\bar{D}, \bar{l}', \bar{l}, \bar{F}$ as well as $\bar{T}$ and $\bar{L}$. Planetary longitudes V_e, M_a and J_u are quoted in Table 10.

Series denoted as U or r yield directly latitude or distance. Series denoted as V yield ΔV. Longitude can be obtained by using formulas (16) or (17). Reference frame is described in Sect. 3. t is dynamical time measured in julian centuries from J 2000.

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Table 9. Leading terms of ELP 2000-85. Coefficients A are measured in arcseconds for V and U and in kilometers for r. Phase φ are measured in degrees. φ is O for (C, O, V), (C, O, U) and $(P, 2, V)$ and 90° for (C, O, r) and $(P, 2, r)$.

D	ℓ'	ℓ	F	A(C,0,V)
0	0	1	0	22639.58578
2	0	-1	0	4586.43830
2	0	0	0	2369.91394
0	0	2	0	769.02571
0	1	0	0	-666.41710
0	0	0	2	-411.59567
2	0	-2	0	211.65555
2	-1	-1	0	205.43582
2	0	1	0	191.95620
2	-1	0	0	164.72851
0	1	-1	0	-147.32129
1	0	0	0	-124.98812
0	1	1	0	-109.38029
2	0	0	-2	55.17705
0	0	1	2	-45.09960
0	0	1	-2	39.53329
4	0	-1	0	38.42983
0	0	3	0	36.12381
4	0	-2	0	30.77257
2	1	-1	0	-28.39708
2	1	0	0	-24.35821
1	0	-1	0	-18.58471
1	1	0	0	17.95446
2	-1	1	0	14.53027
2	0	2	0	14.37970
4	0	0	0	13.89906
2	0	-3	0	13.19406
0	1	-2	0	-9.67905
2	0	-1	2	-9.36586
2	-1	-2	0	8.60553
1	0	1	0	-8.45310
2	-2	0	0	8.05016
0	1	2	0	-7.63015
0	2	0	0	-7.44749
2	-2	-1	0	7.37119
2	0	1	-2	-6.38315
2	0	0	2	-5.74161
4	-1	-1	0	4.37401
0	0	2	2	-3.99761
3	0	-1	0	-3.20969
2	1	1	0	-2.91454
4	-1	-2	0	2.73189
0	2	-1	0	-2.56794
2	2	-1	0	-2.52120
2	1	-2	0	2.48889
2	-1	0	-2	2.14607
4	0	1	0	1.97773
0	0	4	0	1.93368
4	-1	0	0	1.87076
1	0	-2	0	-1.75297
2	1	0	-2	-1.43716
0	0	2	-2	-1.37257
1	1	1	0	1.26182
3	0	-2	0	-1.22412
4	0	-3	0	1.18683
2	-1	2	0	1.17700
0	2	1	0	-1.16169
1	1	-1	0	1.07769
2	0	3	0	1.05950
2	0	1	2	-0.99022
2	0	-4	0	0.94828
2	-2	1	0	0.75168
0	1	-3	0	-0.66938
4	1	-1	0	-0.63521
1	0	2	0	-0.58399
1	0	0	-2	-0.58331
6	0	-2	0	0.57156
2	0	-2	-2	-0.56064
1	-1	0	0	-0.55692
0	1	3	0	-0.54592
2	0	-2	2	-0.53571

D	ℓ'	ℓ	F	A(C,0,U)
0	0	0	1	18461.23868
0	0	1	1	1010.16707
0	0	1	-1	999.69358
2	0	0	-1	623.65243
2	0	-1	1	199.48374
2	0	-1	-1	166.57410
2	0	0	1	117.26069
0	0	2	1	61.91195
2	0	1	-1	33.35720
0	0	2	-1	31.75967
2	-1	0	-1	29.57658
2	0	-2	-1	15.56626
2	0	1	1	15.12155
2	1	0	-1	-12.09414
2	-1	-1	1	8.86814
2	-1	0	1	7.95855
2	-1	-1	-1	7.43455
0	1	-1	-1	-6.73143
4	0	-1	-1	6.57957
0	1	0	1	-6.46007
0	0	0	3	-6.29648
0	1	-1	1	-5.63235
1	0	0	1	-5.36840
0	1	1	1	-5.31127
0	1	1	-1	-5.07591
0	1	0	-1	-4.83961
1	0	0	-1	-4.80574
0	0	3	1	3.98405
4	0	0	-1	3.67446
4	0	-1	1	2.99848
0	0	1	-3	2.79864
4	0	-2	1	2.41388
2	0	0	-3	2.18631
2	0	2	-1	2.14617
2	-1	1	-1	1.76598
2	0	-2	1	-1.62442
0	0	3	-1	1.58130
2	0	2	1	1.51975
2	0	-3	-1	1.51563
2	1	-1	1	-1.31782
2	1	0	1	-1.26427
4	0	0	1	1.19187
2	-1	1	1	1.13461
2	-2	0	-1	1.08578
0	0	1	3	-1.01938
2	1	1	-1	-0.82271
1	1	0	-1	0.80422
1	1	0	1	0.80259
0	1	-2	-1	-0.79319
2	1	-1	-1	-0.79101
1	0	1	1	-0.66741
2	-1	-2	-1	0.65022
0	1	2	1	-0.63881
4	0	-2	-1	0.63371
4	-1	-1	-1	0.59577
1	0	1	-1	-0.58893

D	ℓ'	ℓ	F	A(C,0,r)
0	0	0	0	385000.52899
0	0	1	0	-20905.35504
2	0	-1	0	-3699.11092
2	0	0	0	-2955.96756
0	0	2	0	-569.92512
2	0	-2	0	246.15848
2	-1	0	0	-204.58598
2	0	1	0	-170.73308
2	-1	-1	0	-152.13771
0	1	-1	0	-129.62014
1	0	0	0	108.74270
0	1	1	0	104.75523
0	0	1	-2	79.66056
0	1	0	0	48.88830
4	0	-1	0	-34.78252
2	1	0	0	30.82384
2	1	-1	0	24.20848
0	0	3	0	-23.21043
4	0	-2	0	-21.63634
1	1	0	0	-16.67471
2	0	-3	0	14.40269
2	-1	1	0	-12.83140
4	0	0	0	-11.64995
2	0	2	0	-10.44476
2	0	0	-2	10.32111
2	-1	-2	0	10.05620
2	-2	0	0	-9.88445
2	0	-1	-2	8.75156
1	0	-1	0	-8.37911
0	1	-2	0	-7.00269
1	0	1	0	6.32200
0	1	2	0	5.75085
2	-2	-1	0	-4.95013
0	0	2	-2	-4.42118
2	0	1	-2	4.13111
4	-1	-1	0	-3.95798
3	0	-1	0	3.25824
0	0	0	2	-3.14830
2	1	1	0	2.61641
2	2	-1	0	2.35363
0	2	-1	0	-2.11713
4	-1	-2	0	-1.89704
1	0	-2	0	-1.73853
4	-1	0	0	-1.57139
4	0	1	0	-1.42255
3	0	0	0	-1.41893
0	2	1	0	1.16553
0	0	4	0	-1.11694
0	2	0	0	1.06567

D	ℓ'	ℓ	F	A(P,2,V)
0	1	0	0	0.00487
2	-1	-1	0	-0.00150
2	-1	0	0	-0.00120
0	1	-1	0	0.00108
0	1	1	0	0.00080

D	ℓ'	ℓ	F	A(P,2,r)
2	-1	0	0	0.00149
2	-1	-1	0	0.00111

Ve	T	Ma	Ju	L	D	ℓ'	ℓ	F	φ	A(P,0,V)
18	-16	0	0	0	0	0	-1	0	26.54261	14.24883
0	0	0	0	1	0	0	0	-1	0.00094	7.06304
0	2	0	-2	0	2	0	-1	0	180.11977	1.14307
0	4	-8	3	0	0	0	0	0	285.98707	0.90114
1	-1	0	0	0	0	0	0	0	180.00988	0.82155
18	-16	0	0	0	0	0	-2	0	26.54324	0.78811
18	-16	0	0	0	0	0	0	0	26.54560	0.73930
3	-3	0	0	0	2	0	-1	0	179.98144	0.64371
0	1	0	-1	0	0	0	0	0	1.22890	0.63880
10	-3	0	0	0	0	0	-1	0	333.30551	0.56341

Ve	T	Ma	Ju	L	D	ℓ'	ℓ	F	φ	A(P,0,U)
0	0	0	0	1	0	0	0	0	180.00071	8.04508
0	1	0	0	0	1	0	0	0	276.68007	1.51021
18	-16	0	0	0	0	0	-1	1	26.54287	0.63037
18	-16	0	0	0	0	0	-1	-1	26.54272	0.63014

Ve	T	Ma	Ju	L	D	ℓ'	ℓ	F	φ	A(P,0,r)
0	2	0	-2	0	2	0	-1	0	90.11969	1.05870

Ve	T	Ma	Ju	L	D	ℓ'	ℓ	F	φ	A(P,1,V)
0	0	0	0	0	0	1	0	0	0.00000	1.67680
0	0	0	0	0	2	-1	-1	0	180.00000	0.51642
0	0	0	0	0	2	-1	0	0	180.00000	0.41383
0	0	0	0	0	0	1	-1	0	0.00000	0.37115
0	0	0	0	0	0	1	1	0	0.00000	0.27560
18	-16	0	0	0	0	0	-1	0	114.56550	0.25425
0	0	0	0	0	2	1	-1	0	0.00000	0.07118
0	0	0	0	0	2	1	0	0	0.00000	0.06128
0	0	0	0	0	1	1	0	0	180.00000	0.04516
0	0	0	0	0	2	-2	0	0	180.00000	0.04048
0	0	0	0	0	0	2	0	0	0.00000	0.03747
0	0	0	0	0	2	-2	-1	0	180.00000	0.03707
0	0	0	0	0	2	-1	1	0	180.00000	0.03649
0	0	0	0	0	0	1	-2	0	0.00000	0.02438
0	0	0	0	0	2	-1	-2	0	180.00000	0.02165
0	0	0	0	0	0	1	2	0	0.00000	0.01923

Ve	T	Ma	Ju	L	D	ℓ'	ℓ	F	φ	A(P,1,U)
0	0	0	0	0	2	-1	0	-1	180.00000	0.07430
0	0	0	0	0	2	1	0	-1	0.00000	0.03043
0	0	0	0	0	2	-1	-1	1	180.00000	0.02229
0	0	0	0	0	2	-1	0	1	180.00000	0.01999
0	0	0	0	0	2	-1	-1	-1	180.00000	0.01869
0	0	0	0	0	0	1	-1	-1	0.00000	0.01696
0	0	0	0	0	0	1	0	1	0.00000	0.01623

Ve	T	Ma	Ju	L	D	ℓ'	ℓ	F	φ	A(P,1,r)
0	0	0	0	0	2	-1	0	0	90.00000	0.51395
0	0	0	0	0	2	-1	-1	0	90.00000	0.38245
0	0	0	0	0	0	1	-1	0	90.00000	0.32654
0	0	0	0	0	0	1	1	0	270.00000	0.26396
0	0	0	0	0	0	1	0	0	270.00000	0.12302
0	0	0	0	0	2	1	0	0	270.00000	0.07754
0	0	0	0	0	2	1	-1	0	270.00000	0.06068
0	0	0	0	0	2	-2	0	0	90.00000	0.04970
0	0	0	0	0	1	1	0	0	90.00000	0.04194
0	0	0	0	0	2	-1	1	0	90.00000	0.03[illegible]22

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Table 10. Planetary arguments from VSOP 82. t is measured in julian centuries from J2000. Arguments are measured in arcsec except for constant terms

V_e	$181°58'47''.28305 + 210664136.43355\,t$
M_a	$355°25'59''.78866 + 68905077.59284\,t$
J_u	$34°21'05''.34212 + 10925660.42861\,t$

By using Table 9, one can compute lunar coordinates with a precision of about 20″ from 2000 A.D. to 1500 B.C. for the value $-23''.895/\mathrm{cy}^2$ of the tidal secular acceleration.

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