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AN ANALYSIS OF THE TRANSITS OF MERCURY: 1677–1973

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SUMMARY

About 2400 observations of the universal times of internal contact for the transits of Mercury in the period 1677 to 1973 are compared with the ephemeris times calculated from the gravitational theories of the motions of Mercury and the Earth. The O–C differences in the times are attributed to two causes: the differences between the ephemeris and universal time scales, ΔT (arising from the non-uniformity of the Earth's rate of rotation on its axis) and errors in the values of the orbital elements in the adopted theory of Mercury's motion. In order to separate these two effects, we adopt known estimates of ΔT derived previously from observations of the Moon's motion and solve for corrections to Mercury's orbital elements and a term varying with the square of time. This last term allows for a possible correction to the value of the tidal acceleration in the lunar theory which is implicit in the derivation of the estimates of ΔT from lunar observations. By this method of analysis we find the tidal acceleration of the Moon to be $-26'' \pm 2''$ cy^{-2}, and that the excess of the observed over the Newtonian value of the motion of Mercury's perihelion is $+41''\cdot 9 \pm 0''\cdot 5$ cy^{-1}.

1. INTRODUCTION

Since the transit of Mercury across the Sun's disc in 1677, observers have carefully noted the times at which Mercury's disc apparently touched the limb of the Sun. These observed times were recorded on time-systems based on the diurnal rotation of the Earth, which are related to universal time (Greenwich mean solar time measured from midnight). Variations in the rate of rotation of the Earth cause the universal time scale to depart from uniformity. However, the argument of time, usually called ephemeris time, in the gravitational theories of the motions of the Sun and Mercury, may be regarded as defining a uniform time scale. We may estimate the difference between the ephemeris and universal time scales (ΔT) at the epoch of a transit by subtracting the observed time of the contact of Mercury's disc with the limb of the Sun from the time computed from the theories. Independently, we can derive ΔT from a comparison of the observed times of occultations of stars by the Moon with the computed times on the ephemeris time scale defined by the argument of time in the gravitational theory of the Moon's motion. If the gravitational theories of the Sun, Mercury and the Moon used in the computations were incompatible, owing to deficiencies in their theories, the two sets of deduced values of ΔT would diverge with time. In particular, if the value of the orbital acceleration in the theory of the Moon's motion were in error, we would expect the values of ΔT deduced from lunar occultations to diverge with the square of time from those deduced from the transits of Mercury. In this paper we analyse these two sets of values of ΔT in order to check the provisional value of the orbital acceleration of the Moon's motion in the theory.

The Moon's orbital acceleration has a calculable component ($+14''\cdot28$ cy^{-2}), due to the perturbing effects of the Sun and planets treated as rigid bodies, which is well determined. Another component, due to causes such as the reaction on the lunar motion of the tides raised by the Moon on the Earth, is not well determined. It is this second component which we shall investigate in this paper. We shall refer to it as the 'tidal acceleration'. There may be a third component in the acceleration if the Newtonian constant of gravitation (G) decreases with time. It can be shown that as a result of our method of analysis the deduced values of ΔT would be altered if G were variable, but the value that we derive for the tidal acceleration would not be affected.

In analysing the data on the transits of Mercury, we must recognize that the adopted orbital elements of the theories of the Sun and Mercury may be in error, and that, as a consequence, their computed positions, and hence the estimates of ΔT, will include some contribution from these errors. We use the method of least squares to achieve a partial separation of the effects of errors in the orbital elements and the effects of the variations in the rate of rotation of the Earth. We have used nearly all of the original observations of the times of internal contact of Mercury's disc with the Sun's occurring during the transits of 1677 to that of 1973, giving a total of about 2400 observational equations. Less than 1 per cent of the original observations were rejected in our analysis, which we believe to have two main advantages over all previous analyses of these data. First, we have used all the times for each transit, rather than simply the derived means used by previous authors. Secondly, we have added data from the five transits that have occurred since the last published analysis in 1943.

The main results of this analysis are twofold:

1. We find a value for the tidal acceleration of the Moon close to that found by Spencer Jones and Clemence which is used in the current theories.

2. We obtain a correction to the adopted observed value of the motion of the perihelion of the orbit of Mercury which, when compared with the theoretical one, based on Newtonian theory, gives an excess motion slightly less than that predicted on the basis of Einstein's general theory of relativity.

2. OBSERVATIONS

2.1 *Occurrence of transits*

Since Mercury's orbit is inclined at 7° to the ecliptic, transits occur only at the time of inferior conjunction when the Earth is near the line of nodes of Mercury's orbit. The line of nodes regresses slowly on the ecliptic by about $1\frac{1}{2}$° per century, so the Earth passes through the line of nodes on about the same dates every year: near the beginning of May and November. Mercury passes through its nodal points on those dates about 14 times in a century.

The lines of apsides of the orbits of the Earth and Mercury also move very slowly on the ecliptic, and so the configurations of the orbits are nearly identical for all the transits in the May series, and likewise for those in the November series. The maximum duration for a central transit in May is about 8 hr, whilst in November it is about 6 hr.

2.2 *Timing of contacts*

All the timings of the *internal* contacts with the Sun's disc for the transits of 1677 to 1973 that could be found in astronomical publications were transcribed from the original sources, except for 50 observations which were taken from Newcomb's (1882) work as follows: 1782 (2), 1786 (6), 1789 (6), 1802 (4), 1845 (18) and 1878 (14). About 2400 observations were transcribed for use in the analysis. No timings of the external contacts were transcribed for use because these phenomena were judged to be too indefinite to contribute any worthwhile data to our analysis.

The timings were rounded to the nearest second which, as will be shown later, is considerably less than the smallest value for the standard deviation (11 s) of the timings for any particular transit. Following an inspection of the collected times for each transit, we had little hesitation in rejecting 10 observations which were over 5 min different from the other observations.

There has been an extensive discussion in the astronomical literature of which part of the phenomenon observers actually time. We do not enter into this discussion here, but refer the reader to several places where this subject has been treated (Paul 1876; Newcomb 1882; Clemence & Whittaker 1942; Wittmann 1974). We use all the times as if they correspond to the instants at which the centres of the Sun and Mercury are just separated by the difference of their apparent semi-diameters. Some observers make a distinction between the time of geometrical contact and that of the so-called ' black drop ' effect. But most observers do not report having seen this effect and give the time of contact without qualification. After inspecting the considerable scatter of the collected timings for each transit, we decided to use *all* the timings of internal contacts, irrespective of the observers' descriptive remarks, except when the observer gave separate times for the geometrical contact and the black drop effect. For these observations we selected the times for the black drop because we believe that this is a more definite event than the observed time of geometrical contact which is dependent on the resolution of the telescope employed. If there are systematic differences between these types of observation it will add to the observational scatter of the observations but will not materially affect the principal results of this analysis for the following reason. A systematic delay in timing the second contact may be assumed to be cancelled in the analysis by an equal advance in timing the third contact; and most transits have roughly equal numbers of observations of both contacts. However, such delays would alter the duration of a transit and hence the deduced correction to latitude and adopted semi-diameter.

2.3 *Time-systems and positions of observers*

For transits in the 17th and 18th centuries, the times are usually given in local apparent solar time, and in the 19th century in local mean solar time measured from noon. In some cases sidereal time scales are used.

In general, a change of 1° in an observer's geographical coordinates would change the time of contact by less than 1 s. No observations had to be rejected on this positional criterion alone. But, in order to preserve a precision of 1 s in the conversion of the local times to universal time (i.e. Greenwich mean solar time reckoned from midnight), the longitude is also required with this precision. For observations made at an observatory, where the position was not published with the observations, the longitude, latitude and height were taken, almost exclusively, from *The Nautical Almanac and Astronomical Ephemeris* for the year 1933, where there is a list of the past and current (~1933) positions of the observatories with their corresponding periods of activity. For observations made from places other than observatories we took the positions from the *Connaissance des Temps* for the year 1913, which gives an extensive list of the geographical positions of towns and other places throughout the world. Where the local mean, or apparent, solar time was given, and no accurate longitude could be traced, we adopted the position given by Newcomb (1882). For observations made in the 20th century the times are usually expressed in Greenwich mean solar time (from noon before 1925 and from midnight thereafter). The geographical positions of those observers who only recorded the name of the town or area from which the transit was observed, were taken to the nearest 1′ from the index of *The Times Atlas* (1967). When a sufficiently accurate longitude could not be found, we rejected the observations: a total of 17 observations were rejected for this reason.

2.4 *Publication of observations*

All the observed times of internal contact, reduced where necessary to universal time, the positions of the observers, and the references to the original sources of the data, are to be published elsewhere (Morrison & Ward 1975). We also list there the difference between the computed and observed times of contact which were derived as explained in Section 4.1.

3. REDUCTION OF OBSERVATIONS

The observed times of contact are reduced to universal time (UT), whereas the argument of time in the dynamical theories of the orbital motions of the Earth and Mercury is (nominally) ephemeris time (ET). The maximum difference between ET and UT during the period 1677 to 1973 is less than 3 min and, therefore, UT is a sufficiently good approximation to ET for the calculation of the partial derivatives with respect to time of the parameters to be considered in our analysis. We enter the ephemerides of the Sun and Mercury with the UT of observed contact and calculate the separation of the limbs. The calculated separations are regarded as residuals comprising errors of observation, errors of the orbital elements of the Sun and Mercury, and the differences between the values ET(Sun) and UT for the instants of contact. We use the notation ET(Sun) to indicate explicitly that the time scale concerned is based directly on the fundamental definition of ephemeris time in terms of the Sun's mean longitude; we implicitly assume that in the theory the coefficients in the quadratic expression for the mean longitude of the Sun are absolute constants.

The apparent geocentric right ascension, declination and radius vector of the Sun and Mercury were calculated for each observed time (UT) of contact from a computer program based on that given by Mannino *et al.* (1965) for the evaluation of Newcomb's theories (1891), thus avoiding some of the approximations made by Newcomb in preparing his *Tables* (1898). (A few misprints in the numbers given in Mannino's paper were corrected.) The values of the elements used are given by Newcomb (1895a) in *The elements of the four inner planets and the fundamental constants of astronomy*, hereafter referred to as *Elements*. In particular, we note that the value of precession used in the centennial motions of the perihelia (and nodes) is $5024''\cdot 93$ and there are empirical terms of $+10''\cdot 45$ and $+43''\cdot 37$ in lieu of the general relativity terms $+3''\cdot84$, $+43''\cdot03$ for the Sun (Earth) and Mercury, respectively. A comparison of the spherical coordinates of the Sun evaluated by using Mannino's program with those calculated from Newcomb's *Tables* reveal differences as great as $0''\cdot07$ in longitude which appear to arise from the summation of several periodic terms. There is no evidence of a secular change. Therefore, the basis of our computed positions should not strictly be regarded as Newcomb's *Tables*, but rather as that defined by the evaluation of the series given in Mannino's paper. There is little doubt that the two are sufficiently close for the purposes of this investigation.

If (α', δ') and (α, δ), respectively, denote the topocentric RA and Dec of the Sun and Mercury, derived from the geocentric values for some observed time of contact, then the calculated distance of centres, D, and the position angle of contact, P, reckoned from the north point of the Sun, is computed from the usual relations:

$$\cos D = \sin \delta' \sin \delta + \cos \delta' \cos \delta \cos (\alpha - \alpha')$$

$$\cos P \sin D = \cos \delta' \sin \delta - \sin \delta' \cos \delta \cos (\alpha - \alpha')$$

$$\sin P \sin D = \qquad + \qquad \cos \delta \sin (\alpha - \alpha')$$

The rate of change of D, denoted $\dot{D}$ (in units of $''$/s) is calculated numerically by substituting the rates of change of (α', δ') and (α, δ) in the analytical derivative of the first expression.

The residual separation of the limbs, $\Delta\sigma$, for the time of an internal contact is given by

$$\Delta\sigma = D - (R' - R) \qquad (1)$$

where R' and R are the computed apparent semi-diameters derived from the following adopted values at unit distance:

$$\bar{R}' = 959''\cdot63$$

$$\bar{R} = 3''\cdot37.$$

These residuals are analysed to determine parameters associated with the time scales and corrections to the orbital elements.

4. ANALYSIS OF RESIDUALS

The calculated residual, $\Delta\sigma$, arises from three causes:

1. The difference, ET(Sun) − UT, denoted by ΔT(Sun);
2. Errors in the adopted values of the orbital elements in the theories of the Sun and Mercury;
3. Errors in timing.

First, we analyse the residuals for the effects arising from errors in timing.

Fig. 1 is a histogram of the residuals ($\Delta\sigma$) for the observations of contact 2 in the transit of 1878. The normal frequency curve for the expected distribution of errors using a mean and standard deviation estimated in the usual way from the residuals is also shown. The histogram appears to be slightly asymmetrical. To test for the goodness-of-fit of the residuals to the normal curve we calculated the value of χ^2 and found it to be 9 by grouping the residuals into 10 classes in the range $-0\cdot9$ to $+0\cdot9$. If the observational errors followed the normal distribution there would be about one chance in ten of χ^2 exceeding 12 for 7 degrees of freedom.

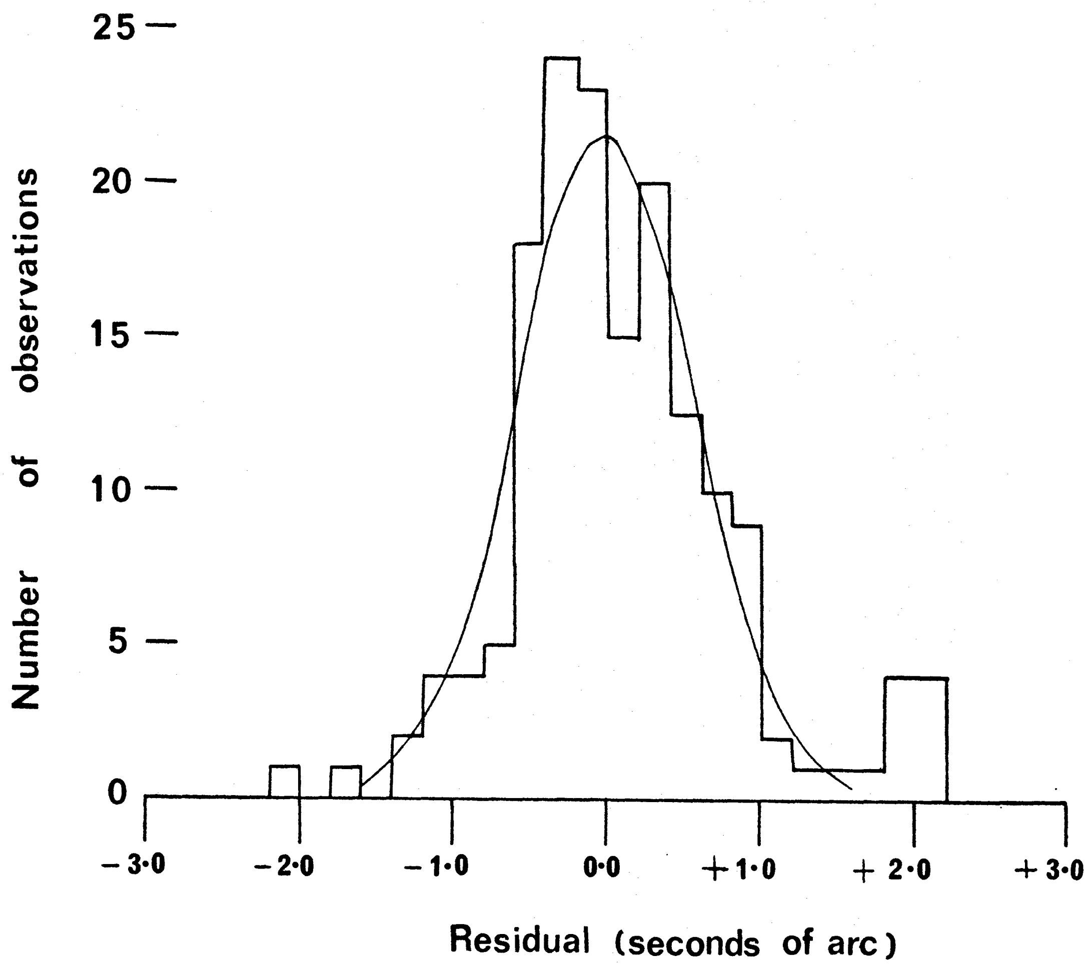

FIG. 1. *Histogram of the residuals of contact 2 for the transit of* 1878.

Hence, we conclude that there is not strong evidence to suggest that the errors depart substantially from the normal distribution. Some of the histograms of the residuals for the contacts of other transits also display a similar amount of asymmetry to that shown in Fig. 1, but collectively they show no preferred direction for the bias. We therefore proceed on the assumption that when all the observations of the transits are combined, the errors in timing will tend to follow the normal law.

A more serious source of bias in the data is that all the observers may tend to be late in timing contact 2 and early in timing contact 3, or vice versa. (The fact that the mean is zero in Fig. 1 is accidental.) Whether or not an observer is likely to be early or late in timing a particular contact will depend on many factors which are alluded to in Section 2.2. In general, this delay or anticipation in timing will be equal and opposite for contacts 2 and 3 and hence tend to cancel in our analysis for determining values of the parameters associated with the time scales and the orbital elements in longitude. But these systematic effects in timing could alter the durations of transits and hence affect the solution for corrections to the orbital elements in latitude. We investigate the size of these effects in Section 5.3.

Since observations of the transits lead to the determination of corrections at two points in Mercury's orbit, the errors in the adopted elements will give rise to constant and secular contributions to $\Delta\sigma$, and not periodic ones.

The difference, ΔT(Sun), is believed to consist of a secular change (due primarily to the more or less uniform effects of tidal friction), irregular changes over decades (probably due to core–mantle interaction) and quasi-periodic changes of about a year or less (due primarily to changes in the circulation of the atmosphere).

In this paper we use two methods to find the contributions in the residuals, $\Delta\sigma$, due to ΔT(Sun) and the corrections to the orbital elements. In Section 4.1 we neglect the relatively small contributions due to errors in the elements, and attribute the whole of the residual to the difference between the time scales, ΔT(Sun). It is recognized that, by neglecting corrections to the elements, the derived values of ΔT(Sun) will be altered by a small constant and linear term, but their general behaviour with time will otherwise be unaffected. In Section 4.3 we adopt values of ΔT(Moon) from lunar observations and substitute these for ΔT(Sun). We assume that ΔT(Sun) and ΔT(Moon) differ only by a quadratic expression in time in which the constant and linear term are already known. Equations of condition are then set up with unknowns for corrections to the orbital elements and an expression varying with the square of time arising from the difference between ΔT(Sun) and ΔT(Moon).

4.1 *Solution for values of ΔT(Sun)*

The residuals, $\Delta\sigma$, were converted to time residuals, Δt, for the 2341 observations which had not been rejected for some reason in Section 2, by using the expression

$$\Delta t = -\Delta\sigma/\dot{D}.$$

Those lying in the range -80 s to $+120$ s are plotted in Fig. 2: 14 values lie outside this range.

There are sufficient numbers of observations for all the transits beginning with the year 1789 to make a reliable estimate of the standard deviation of the error distribution for each transit. In calculating the standard deviations we excluded 11 observations giving values of ΔT(Sun) outside the range ± 120 s since such values appear to correspond to gross errors in timing or recording. We recall that Mercury moves through its own diameter in about 1 min, so that after 2 min it is normally well clear of the Sun's limb, except near grazing, as in the transits of 1782 and 1957.

Estimates of ΔT(Sun) are formed by taking the means of the time residuals for each transit:

$$\Delta T(\mathrm{Sun}) = \overline{\Delta t}.$$

These estimates are given in Table I together with their standard errors and the standard deviations of the time residuals from their means.

Points lying outside the limit of ± 2 sd are circled in Fig. 2. Before the transit of 1789 we adopted an approximate mean of zero and acceptance limits of ± 1 min for all the transits. Points lying outside these limits are also circled in Fig. 2. Of the total of 2327 points plotted in Fig. 2, 146 lie outside our acceptance limits. Fig. 3 is a plot of the mean values of ΔT(Sun) and their standard errors for each transit beginning with 1789 calculated *after* the exclusion of the points shown circled in Fig. 2.

4.2 *Preliminary comparison of ΔT(Sun) with ΔT(Moon)*

Fig. 4 shows the annual mean values of the difference,

$$\Delta T(\mathrm{Moon}) = \mathrm{ET(Moon)} - \mathrm{UT},$$

for the years 1663 to 1972 deduced from observations (mainly occultations) of the Moon. The values are taken from the following sources with minor corrections as indicated:

1663–1860 Analysis by Martin (1969), with 1·25 s added to correspond to the application of a correction to the right ascensions of the stars to reduce

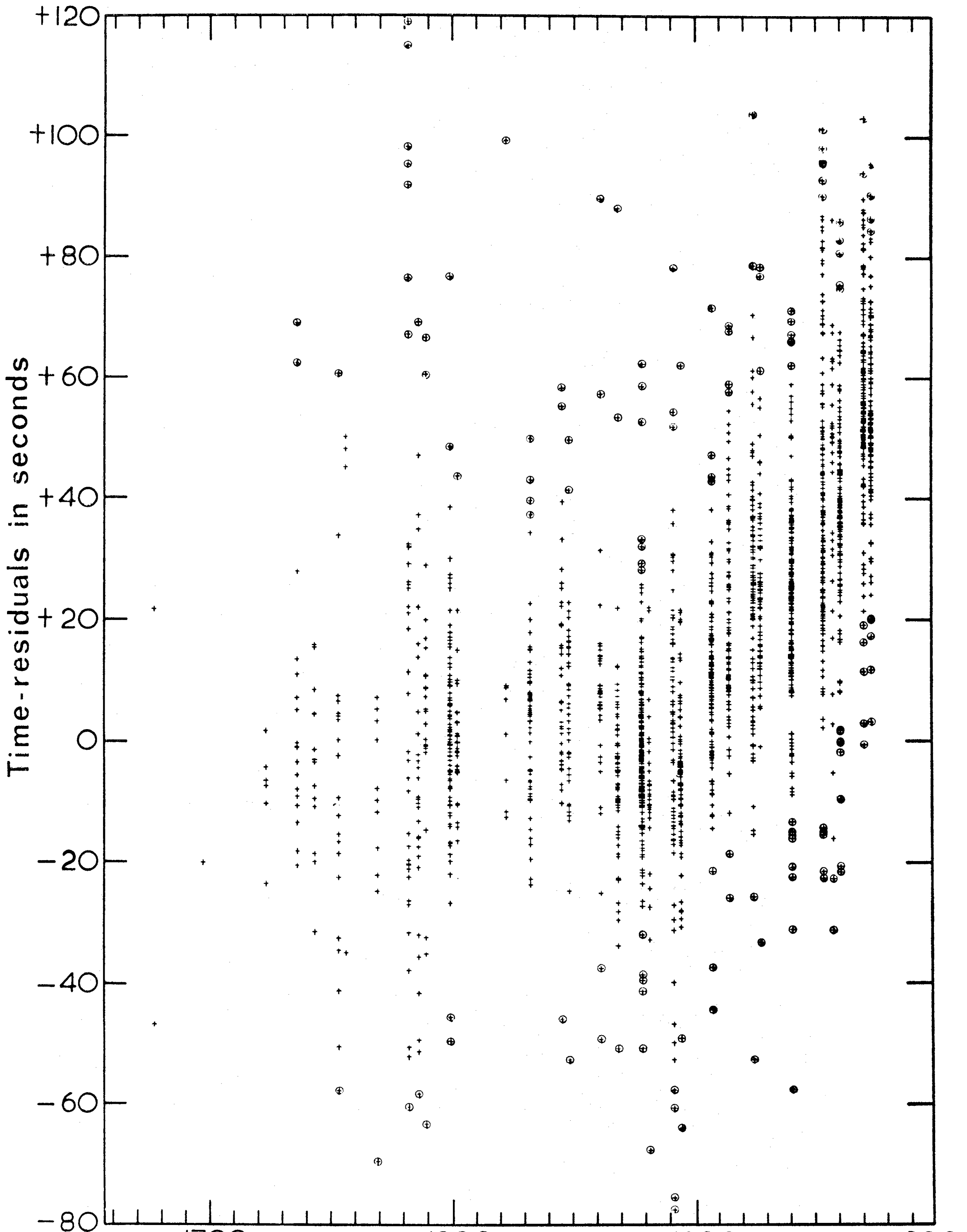

FIG. 2. *Time residuals of internal contacts for the transits of 1677–1973.*

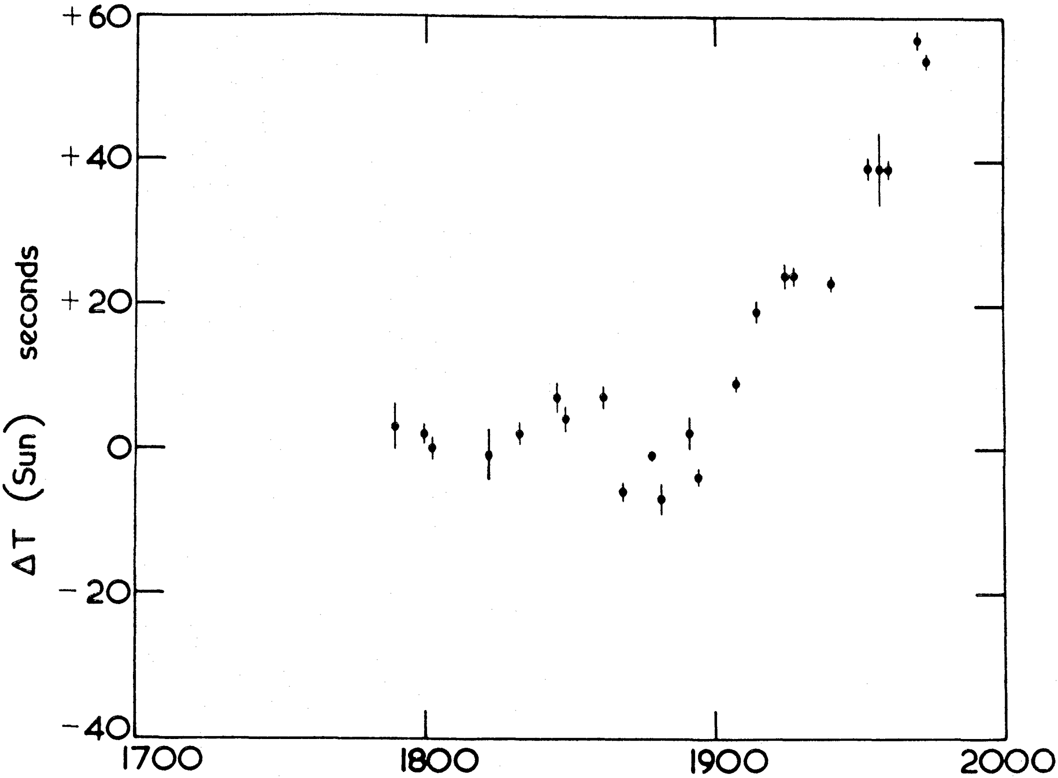

Fig. 3. *Estimates of ΔT(Sun) for transits beginning with that of 1789. They are the means of the time residuals shown in Fig. 2. The half-length of each bar is one standard error.*

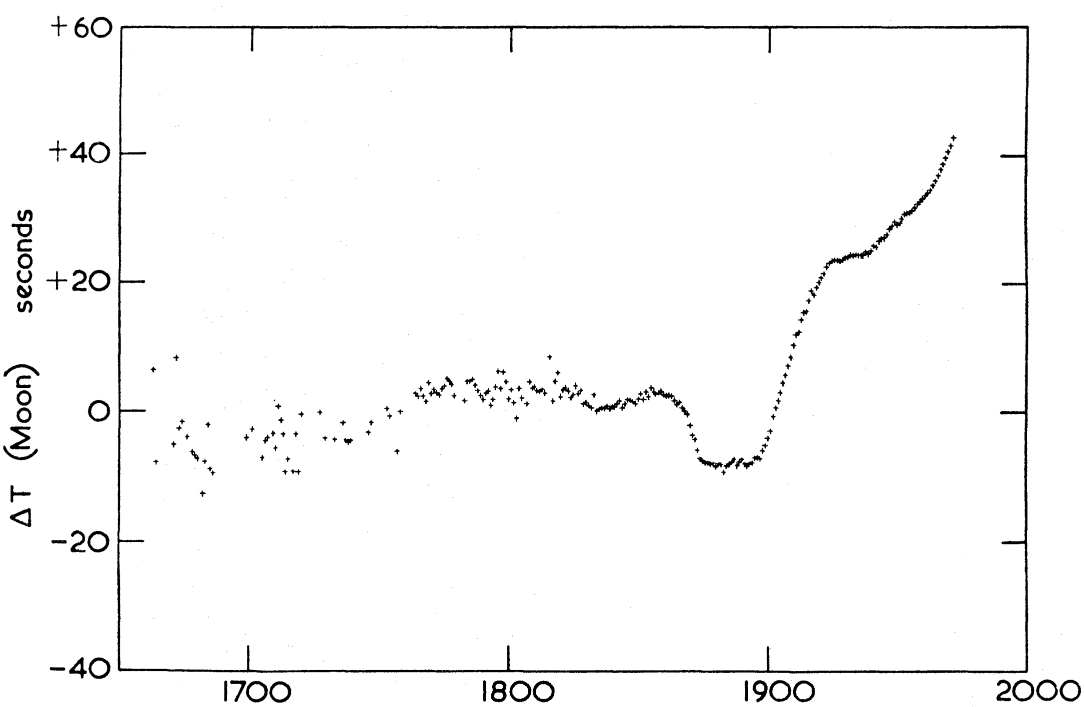

Fig. 4. *Annual mean values of ΔT(Moon) derived mainly from lunar occultations. The analytical theory of the Moon's motion used in their derivation included the Jones–Clemence value of $-22''\cdot 44$ cy^{-2} for the tidal acceleration.*

TABLE I

Mean values, their standard errors, and the standard deviations of the time residuals, Δt, for each transit beginning with that of 1789. The means are estimates of ΔT(Sun). (Units: seconds)

Year	Mean	se	sd	Year	Mean	se	sd
1789	+5	5	26	1894	−5	1	15
1799	+1	2	19	1907	+10	1	14
1802	+1	2	11	1914	+21	2	18
1822	+12	12	34	1924	+25	2	24
1832	+5	2	15	1927	+25	2	16
1845	+8	3	19	1940	+23	1	18
1848	+5	3	17	1953	+40	2	24
1861	+8	3	20	1957	+38	7	28
1868	−5	2	18	1960	+38	1	18
1878	0	1	14	1970	+56	1	18
1881	−9	3	16	1973	+54	1	15
1891	0	3	27				

them from the catalogue equinox of the FK4 to that of Newcomb (Newcomb's equinox).

1861–1949 Discussion by Brouwer (1952), with 1·00 s added to the values after 1922 in order to reduce them to Newcomb's equinox.

1950–1954 *Astronomical Ephemeris* for 1971, page vii.

1955–1972 Calculated from the relation

$$\mathrm{ET} - \mathrm{UT2} = (\mathrm{TAI} - \mathrm{UT2}) + 32{\cdot}24\ \mathrm{s}$$

where the values of TAI − UT2 were taken on July 01 of each year from the circulars in the D series issued by the Bureau International de l'Heure. The constant term 32·24 s is based on an analysis of the lunar occultation data for the period 1960–66 and allows for a correction of 1·34 s in order to refer the values to Newcomb's equinox.

All the values of ΔT(Moon) taken from these sources were deduced from observations using the same expression for the mean longitude of the Moon. The expression was derived from that used in Brown's (1919) *Tables of the motion of the Moon* by removing his empirical term in longitude, thus reducing the motion to gravitational theory. The amended expression includes the term $+7''{\cdot}14T^2$ in mean longitude, which arises from planetary perturbations. All the values of the lunar acceleration (twice the coefficient of T^2 in mean longitude) discussed in this paper are in addition to this term. The following correction, derived from results of Spencer Jones (1939) by Clemence (1948), was then added to the gravitational expression in an attempt to bring the time scale of the lunar ephemeris, ET(Moon), into agreement with that of the solar ephemeris, ET(Sun):

$$-8''{\cdot}72 \quad -26''{\cdot}74T \quad -11''{\cdot}22T^2,$$

where T denotes time in centuries measured from the epoch 1900 January 0·5 UT. The last term in this expression allows for a tidal acceleration in longitude of $-22''{\cdot}44$ cy^{-2}, which will be referred to hereafter as the Jones–Clemence value. The small differences between the other revisions of Brown's theory, such as the introduction of revised values for the astronomical constants, have a negligible effect on the deduced values of ΔT(Moon).

A visual comparison of Figs 3 and 4 reveals immediately the similar behaviour of ΔT(Sun) and ΔT(Moon). This is to be expected since it was the close agreement of the corresponding fluctuations in the longitudes of the Sun, Mercury and the Moon which Spencer Jones (1939) used to demonstrate conclusively that the Earth's rate of rotation is variable.

We now proceed to a more detailed comparison of ET(Sun) with ET(Moon). We recognize that they may differ by a quadratic expression in T resulting from possible errors in the empirical relation given by Clemence above. The constant and linear term in that expression cannot be re-determined without a discussion of observations of the Sun and Moon. We do not propose to undertake this here and so we adopt these values in the present analysis. However, we do propose to solve for a correction to the coefficient of T^2. We also allow for corrections to the values of the elements in Newcomb's theories of Mercury and the Sun, except that we do not consider corrections to the coefficients in the mean longitude of the Sun since they are the basis for the definition of ET(Sun). With these considerations, we develop the observational equation of condition for the analysis of the residuals computed by relation (1).

4.3 *Observational equation of condition*

Fig. 5 shows the apparent configuration on the celestial sphere of Mercury (M) and the Sun (S) at second contact during a November transit when Mercury's motion in right ascension is retrograde. The configuration is similar for a May transit except that the transit occurs near the descending node. The position angle, Q, of the line of centres is measured from the normal to the ecliptic at S. It is calculated from the position angle P, measured from the north point, using the relation

$$Q = P + \eta$$

where η is given by

$$\sin \eta = \cos \alpha' \sin \epsilon \quad [-90° \leqslant \eta \leqslant +90°]$$

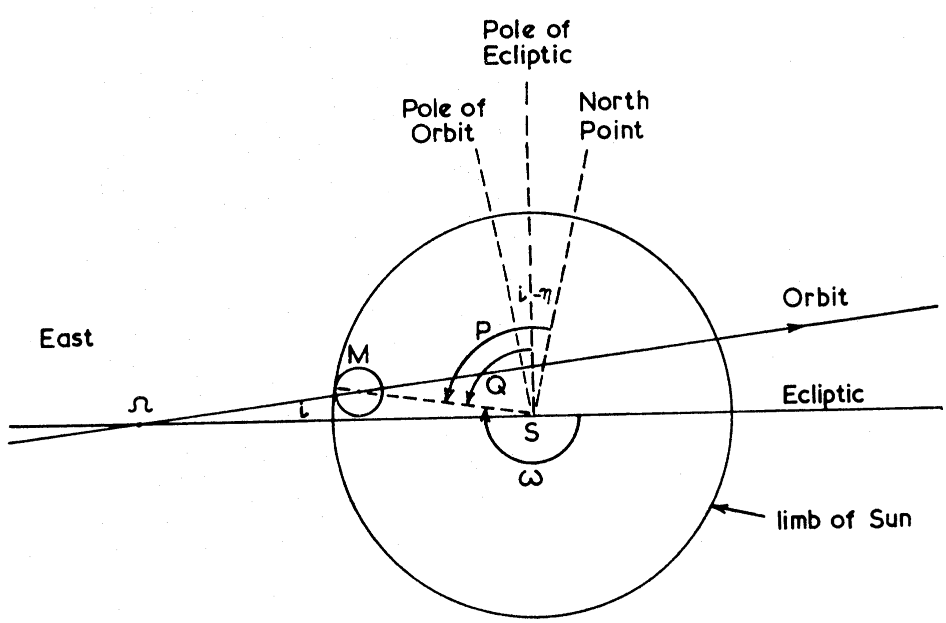

FIG. 5. *Diagrammatical representation of a November transit at the time of internal contact* (2) *of Mercury* (M) *and the Sun* (S) *occurring near the ascending node of Mercury's orbit. Mercury's motion in right ascension is retrograde.*

and ϵ is the obliquity of the ecliptic. In the plane approximation the angle between the normals to the ecliptic and orbit at S can be taken equal to i with an error not exceeding 7×10^{-5} rad. Newcomb (1882, pp. 440–455) gives a full account of the derivation of the partial derivatives for changes in the heliocentric orbital elements of Mercury and the Earth. We see no purpose in giving that derivation again here and so we use his equation of condition with some modifications:

$$+\sin(Q-i)\,V+\sin(Q+i)\,W\pm\cos(Q\mp i)\,N-\frac{1}{r}\,\Delta\bar{R}-\frac{d}{r}\,\dot{D}\,\Delta T(\mathrm{Sun}) = \frac{d}{r}\,\Delta\sigma. \tag{2}$$

The unknowns, V, W and N, have the same meaning as in Newcomb's work. They are linear functions of corrections to the elements of the orbits of Mercury and the Earth for the epoch 1900·0 which are introduced in this form because observations of transits only provide data for two points of the orbits: V is the average correction in longitude for all the November transits and W for the May transits; N is principally the correction to the longitude of the node of Mercury's orbit at both November and May transits. The upper sign refers to the November transits and the lower one to the May transits. We note that Newcomb (1882, p. 448, equation 3′) has omitted the minus sign in front of his coefficient for N in the May transits. This error was carried through to his equations of condition and therefore his solution for the correction to the longitude of the node is erroneous. The approximations introduced by Newcomb in deriving these functions of the elements will not materially affect the results of our analysis. The numerical expressions for the functions are given in Section 5.2 where we discuss the results. These unknowns are assumed to be linear functions of time, e.g. $V = V_0+\dot{V}T$, where V_0 is the value for the epoch 1900·0 and $\dot{V}$ is the centennial rate of change. The position angle, Q, in the coefficients is equal to $270^\circ-\omega$, where ω is the angle denoted by Newcomb shown in Fig. 5.

We do not include an unknown for the effect of a correction to the adopted mass of Venus in the theory of Mercury as Newcomb did, but, instead, prefer to allow for its effect afterwards by adjusting the results (principally the motion of the perihelion) for the value of the mass recently determined from space probes. Separate corrections to the semi-diameters of Mercury and the Sun cannot be reliably determined because of the close correlation in their coefficients. We only consider an unknown, $\Delta\bar{R}$, for the semi-diameter of Mercury at unit distance, but this could be regarded alternatively as a correction of $-1{\cdot}6\,\Delta\bar{R}$ to the semi-diameter of the Sun. This correction has little meaning physically, due to the particular nature of transit observations. It is introduced to prevent any contribution from it in the residuals being absorbed by the other unknowns. In its coefficient, r is the heliocentric distance of Mercury in astronomical units.

The last unknown in the observational equation (2) allows for the fact that in our reduction the positions of Mercury and the Sun were computed from their theories using the UT of observation instead of the ET(Sun). The factor, d/r, is the ratio of the geocentric and heliocentric distances of Mercury: it reduces the O–C, $\Delta\sigma$, computed geocentrically (strictly, topocentrically), to the heliocentric distance of Mercury.

4.4 *Solution of observational equation*

The observational equation (2) cannot be solved in its present form because the constant and secular parts of ΔT(Sun) are highly correlated with V_0 and $\dot{V}$

(or W_0 and $\dot{W}$), respectively. This difficulty is avoided by substituting the values of ΔT(Moon) for ΔT(Sun) in equation (2) and introducing instead an unknown varying as T^2 for the reasons given in Section 4.2. By making this substitution we are assuming that the constant and linear parts of the quadratic expression given by Clemence in Section 4.2, which relate the origin and rate of the two time scales, ET(Sun) and ET(Moon), are correct, and that the coefficients in the expression for the mean longitude of the Sun in Newcomb's theory are being regarded as basic constants. With these conditional prerequisites, we regard those parts of the functions V_0, $\dot{V}$, W_0 and $\dot{W}$ which are dependent on corrections to the origin and mean motion in longitude as arising only from corrections to Mercury's elements.

It is fundamental to our analysis that none of the other unknowns in longitude in equation (2) contains a term varying as T^2; that is, there are no accelerations of significant size in the orbital motions of Mercury or the Earth, other than those treated by Newcomb in his gravitational theories. If the gravitational constant, G, were decreasing with time, as considered by Hoyle (1972), this would lead to the relation

$$\frac{2\dot{G}}{G} = \frac{\dot{n}}{n},$$

where n is the mean motion. This would, therefore, introduce an acceleration, $\dot{n}$, in the orbital motion which was proportional to the mean motion. By neglecting this acceleration in our analysis, the deduced values of ΔT(Sun) and ΔT(Moon) would *both* be in error by the amount $(\dot{G}/G)\,T^2$. In our equation of condition we are solving for a *differential* correction, varying as T^2, between the time scales ET(Sun) and ET(Moon), and therefore any possible contribution from $\dot{G}$ would cancel. For this reason, we interpret our solution for the term in T^2 as arising solely from the tidal acceleration of the Moon.

In equation (2), we move the term in ΔT(Sun) to the RHS and substitute for ΔT(Sun) the adopted values of ΔT(Moon) listed in Table II; and on the LHS we introduce the term $+1{\cdot}821 \times \frac{1}{2}kT^2$. The factor, 1·821, is the reciprocal of the Moon's mean motion in units of s/″, and k is the correction to the Jones–Clemence value of the Moon's tidal acceleration. The equation of condition then becomes

$$+\sin(Q-i)(V_0+\dot{V}T)+\sin(Q+i)(W_0+\dot{W}T)\pm\cos(Q\mp i)(N_0+\dot{N}T) -\frac{1}{r}\,\Delta\bar{R}+0{\cdot}911\,\frac{d}{r}\,\dot{D}kT^2 = \frac{d}{r}\,[\Delta\sigma+\dot{D}\,\Delta T(\text{Moon})]. \qquad (3)$$

TABLE II

Values of ΔT(Moon) from occultations, using the Jones–Clemence value of $-22''{\cdot}44$ cy^{-2} for the tidal acceleration. (Units: seconds)

Year	ΔT	Year	ΔT	Year	ΔT
1677	−5·4	1799	+3·1	1894	−7·7
1697	−5·4	1802	+3·1	1907	+5·9
1723	−3·0	1822	+3·1	1914	+15·6
1736	−3·0	1832	+1·2	1924	+23·4
1743	−2·6	1845	+1·2	1927	+23·9
1753	−0·7	1848	+1·9	1940	+25·2
1756	−0·2	1861	+2·7	1953	+31·0
1769	+3·1	1868	+0·3	1957	+32·0
1782	+3·1	1878	−7·9	1960	+33·4
1786	+2·9	1881	−7·9	1970	+40·7
1789	+2·9	1891	−7·9	1973	+44·0

TABLE III

Solutions for the unknowns in equation (3): k is the correction to the Jones–Clemence value of $-22''.44$ cy^{-2} for the tidal acceleration of the Moon. The uncertainties are standard errors

Transits	Observations	V_0 (″)	$\dot{V}$ (″ cy^{-1})	W_0 (″)	$\dot{W}$ (″ cy^{-1})	N_0 (″)	$\dot{N}$ (″ cy^{-1})	$\bar{R}$ (″)	k (″ cy^{-2})	Σ (residual)^2 Variance
1677–1973 (33)	1. All (2339)	+0·32	+0·92	+0·50	+0·60	−0·70	−0·10	+0·05	−1·8	17980
		±0·12	±0·19	±0·11	±0·18	±0·11	±0·18	±0·02	±1·0	7·7
	2. Within limits (see Section 4.1) (2180)	+0·17	+1·17	+0·39	+0·76	−0·68	−0·02	+0·02	−3·3	7865
		±0·08	±0·13	±0·08	±0·12	±0·08	±0·12	±0·01	±0·7	3·6
1789–1973 (23)	3. All (2187)	+0·22	+0·78	+0·44	+0·54	−0·70	−0·18	+0·04	−5·0	15792
		±0·13	±0·20	±0·11	±0·19	±0·12	±0·26	±0·02	±1·6	7·2
	4. Within limits (2169)	+0·09	+1·10	+0·35	+0·71	−0·68	−0·03	+0·02	−5·5	7028
		±0·09	±0·14	±0·08	±0·13	±0·08	±0·18	±0·01	±1·1	3·3

The observational equations in eight unknowns were solved by the method of least-squares: first, with all the observations (2341); and, secondly, without the observations producing residuals lying outside the limits discussed in Section 4.1 and shown as circled points in Fig. 2. These observations comprised about 6 per cent of the total. The two solutions for the unknowns and their standard errors are displayed in Table III. The sum of the squares of the residuals from solution 2 was less than half that from solution 1. We also made corresponding solutions by excluding all the transits before 1789. (Table III, solutions 3 and 4.) It can be seen from Fig. 2 that there are considerably fewer observations (152) for these transits and we were not able to compute rigorous limits for rejection of observations by statistical methods alone.

The solution is not altered significantly by rejecting those observations with residuals lying outside the restrictive limits derived in Section 4.1. But it seems to be important in the solution for k whether or not we include observations of the transits before 1789. As expected, by shortening the period of the observations, we increased the correlations between the unknowns and weakened the solution for the acceleration. We can find no good reason to reject *all* the observations before 1789. Only six of the original 154 made in that period fall outside the range of Fig. 2. Except for the grazing transit of 1782, they are no more disparate than the observations after 1789, though fewer in number. We adopt solution 2 in our discussion of the corrections to the orbital elements.

5. DISCUSSION OF RESULTS

5.1 *Solution for tidal acceleration of Moon*

The values of k in Table III, added to the provisional acceleration of $-22''\cdot44$ cy^{-2}, give the results for the tidal acceleration shown in the third column of Table IV. We have tested that our solution for the acceleration is independent of our initial approximation. It is important to verify this because the separation of the unknown in T^2 from the other unknowns in equation (3) may not be as complete as is desired, although none of the correlation factors between the unknowns was greater than 0·5. The test was made by starting with the value $-42''\cdot44 \pm 6''$ cy^{-2}, found by Morrison (1973) for the acceleration and substituting his values of ΔT(Moon), derived using this acceleration in the observational equation (3), as described in Section 4.4. The solutions for k were close to the previous ones: the resulting values of the acceleration are given in Table IV.

We have investigated the effect on the residuals of holding the value of the acceleration at $-42''\cdot44$ cy^{-2} ($k = -20''$ cy^{-2}) and solving for the remaining seven unknowns in Table III. The resulting sum of squares of the residuals was increased

TABLE IV

Solutions and standard errors for the tidal acceleration of the Moon. (Units: $''$ cy^{-2})

		Initial approximation	
		−22·44	−42·44
Transits	Observations	(Jones–Clemence)	(Morrison)
1677–1973 (33)	1. All	−24·2 ± 1·0	−24·5 ± 1·0
	2. Within limits	−25·7 ± 0·7	−26·0 ± 0·7
1789–1973 (23)	3. All	−27·4 ± 1·6	−28·1 ± 1·6
	4. Within limits	−27·9 ± 1·1	−28·5 ± 1·1

from 7865 in solution 2 of Table III to 9922. This increase in the sum of squares for the loss of one degree of freedom is certainly significant. But this does not exclude the possibility that systematic biases in the timing of the contacts may have distorted our result. We show later, when discussing the correction to the motion of the node, that there are biases in the data which probably affect that result; but it is very difficult to imagine why these biases should tend to depart as T^2 over the 300-yr period. The technique used in observing transits has not changed substantially over the period and any increase with time in the resolving power of telescopes would tend to alter the timing of contacts 2 and 3 by equal and opposite amounts, thus changing the observed duration but not the overall mean time. If the true value of k were $-20''$ cy^{-2}, this would imply a bias in timing of the transits of $+18$ s T^2, which seems very unlikely even from a cursory inspection of Figs 2 and 3. These data cannot be reconciled with a value of the tidal acceleration of the Moon as great as $-42''$ cy^{-2} and possible changes to the values of the orbital elements of Mercury and the Earth do not alter this conclusion.

In view of the discordance of the results in Table IV, we think that the standard error of $\pm 0{\cdot}7$ for solution 2 is underestimated, and that the most likely result from all the data for the Moon's tidal acceleration is in the range $-26 \pm 2''$ cy^{-2}.

5.2 *Corrections to Mercury's orbital elements*

The six unknowns, V_0, $\dot{V}$, W_0, $\dot{W}$, N_0 and $\dot{N}$, shown in Table III, are functions of corrections to values of the orbital elements of Mercury and the Sun. The expressions given by Newcomb (1882, pp. 447, 448), and adjusted slightly (Clemence 1943, p. 53) are:

$$V_0 = +1{\cdot}49\,\Delta L_0 - 0{\cdot}49\,\Delta\pi_0 - 1{\cdot}08\,\Delta e_0 + 1{\cdot}19\,e'\,\Delta\pi_0' + 1{\cdot}58\,\Delta e_0' \qquad (4)$$

$$W_0 = +0{\cdot}72\,\Delta L_0 + 0{\cdot}28\,\Delta\pi_0 + 0{\cdot}90\,\Delta e_0 - 1{\cdot}11\,e'\,\Delta\pi_0' - 1{\cdot}62\,\Delta e_0' \qquad (5)$$

$$N_0 = \sin i\,(\Delta\theta_0 - \Delta l_0'). \qquad (6)$$

These corrections have the following meaning:

ΔL_0 is the correction to the mean longitude of Mercury,
$\Delta\pi_0$ is the correction to the longitude of the perihelion of Mercury's orbit,
$\Delta\theta_0$ is the correction to the longitude of the node of Mercury's orbit,
Δe_0 is the correction to the eccentricity of Mercury's orbit expressed in seconds of arc,
$\Delta e_0'$ is the correction to the eccentricity of Earth's orbit expressed in seconds of arc,
$e'\,\Delta\pi_0'$ is the correction to the longitude of the Earth's perihelion multiplied by the eccentricity of the Earth's orbit (0·01675),
$\Delta l_0'$ is the correction to the true longitude of the Earth: it is related to the other corrections by,

$$\Delta l_0' = -2e'\,\Delta\pi_0' \cos(L' - \pi') + 2\Delta e_0' \sin(L' - \pi'), \qquad (7)$$

where L' is the mean longitude of the Earth.

In the expressions for V_0, W_0 and $\Delta l_0'$ we have omitted a correction for the mean longitude of the Earth (Sun) because in our analysis we have adopted Newcomb's expression as the basis of the definition of ET(Sun). The equations for the centennial variation of the elements measured from the epoch 1900·0 have the same coefficients as above.

5.2.1 *Corrections to the constant parts of Mercury's elements.* We adopt results from meridian circle observations in order to eliminate three of the unknowns in equations (4) and (5). From a discussion of meridian circle observations of Mercury made in the period 1765–1937, Clemence (1943, p. 49) found the following correction to the linear combination of three of the elements:

$$\Delta e_0 + 1{\cdot}35 e' \, \Delta\pi_0' - 3{\cdot}00 \Delta e_0' = -0''{\cdot}40 \pm 0''{\cdot}04 \text{ (pe)}. \tag{8}$$

Adams & Scott (1968) analysed meridian circle observations of the Sun made between 1956 and 1962 and found the following corrections to the Sun's elements (p. 318, Table 16):

$$e' \, \Delta\pi_0' = -0''{\cdot}19 \pm 0''{\cdot}01 \text{ (pe)}$$

$$\Delta e_0' = -0''{\cdot}15 \pm 0''{\cdot}01 \text{ (pe)}.$$

We insert these values in relation (8) and, converting the probable errors to standard errors, we find

$$\Delta e_0 = -0''{\cdot}59 \pm 0''{\cdot}08.$$

With these values for $e' \, \Delta\pi_0'$, $\Delta e_0'$ and Δe_0 and our solutions for V_0 and W_0 (Table III, solution 2) we find from relations (4) and (5)

$$\Delta L_0 = +0''{\cdot}30 \pm 0''{\cdot}08$$

$$\Delta\pi_0 = +0''{\cdot}91 \pm 0''{\cdot}24.$$

We cannot find the correction to the node, $\Delta\theta_0$, from relation (6) without first eliminating $\Delta l_0'$. We see from relation (7) that $\Delta l_0'$ has equal and opposite values in May and November. If we insert the corrections to $e' \, \Delta\pi_0'$ and $\Delta e_0'$ given above in relation (7), we find

$$\Delta l_0' = +0''{\cdot}03 \text{ in May}$$

and

$$\Delta l_0' = -0''{\cdot}03 \text{ in November.}$$

As we would expect, the correction to the true longitude of the Earth is comparatively small and we therefore neglect it in the expression for N_0. Taking the value of N_0 from Table III (solution 2) and $\sin i = 0{\cdot}122$, we find from relation (6)

$$\Delta\theta_0 = -5''{\cdot}7 \pm 0''{\cdot}6.$$

5.2.2 *Corrections to the secular variations of Mercury's elements.* Apart from the value of the general precession of the Earth's equator on the ecliptic and the empirical terms in the perihelia, Newcomb's values for the secular variations of the elements of Mercury and the Sun are derived from theory, and not observation. The component parts of these theoretical values are directly proportional to the masses of the disturbing planets. We now have more accurate values of the masses than were available to Newcomb and we therefore compute corrections to Newcomb's values of the secular variations due to the fractional changes in the masses. This computation is facilitated by the use of tables prepared by Newcomb (1895b), which set out the contributions to the secular variations due to the action of each planet. In Table V we list the reciprocals of the values of the masses used by Newcomb and those provisionally recommended by the IAU Commission 4

TABLE V

Reciprocal masses of planets

	Newcomb	IAU Commission 4	Fractional change in mass
Mercury	6000000	6023600	−0·00392
Venus	408000	408523·5	−0·00128
Earth + Moon	329390	328900·2	+0·00149
Mars	3093500	3098710	−0·00168
Jupiter	1047·35	1047·355	0·00000

Working Meeting on Astronomical Constants and Ephemerides, Washington 1974. The effects of the changes in the masses of the planets beyond Jupiter are insignificantly small for the purposes of this paper. Multiplying the terms given by Newcomb (1895b, pp. 375, 377) by the fractional changes in masses listed in Table V and adding the contributions, we find the following corrections to Newcomb's values of the secular variation of the elements:

Changes to centennial variation of the elements resulting from the revision of planetary masses in Table V

$$\begin{aligned} \Delta\dot{e} &= -0''\cdot002\ \mathrm{cy}^{-1} \\ \Delta\dot{\pi} &= -0''\cdot222\ \mathrm{cy}^{-1} \\ \sin i\ \Delta\dot{\theta} &= +0''\cdot013\ \mathrm{cy}^{-1} \\ \Delta\dot{e}' &= +0''\cdot001\ \mathrm{cy}^{-1} \\ e'\ \Delta\dot{\pi}' &= -0''\cdot010\ \mathrm{cy}^{-1} \\ \Delta(\kappa''\sin L'') &= -0''\cdot010\ \mathrm{cy}^{-1} \\ \Delta(\kappa''\cos L'') &= +0''\cdot038\ \mathrm{cy}^{-1}. \end{aligned}$$

The symbols κ'' and L'' have the same meaning as in Newcomb's work. They define the slow rotation of the ecliptic by the amount κ'' about a slowly-moving diameter in longitude L'' due to the gravitational action of the planets on the Earth: it is usually called the planetary precession. The value of the expression

$$-\Delta(\kappa''\sin L'')\cot\epsilon,$$

where ϵ is the obliquity of the ecliptic, is the correction to Newcomb's value of the general precession due to changes in the masses of the planets. Substituting the value above for $\Delta(\kappa''\sin L'')$, we find it is equal to $+0''\cdot02$. This correction is very small relative to the uncertainty associated with the value of luni-solar precession ($\gtrsim \pm 0''\cdot1$) and so we shall ignore it when discussing corrections to Newcomb's value of general precession.

We now correct Newcomb's value for the motion of the Earth's perihelion by removing his empirical term of $+10''\cdot45\ \mathrm{cy}^{-1}$ and replacing it by $+3''\cdot84\ \mathrm{cy}^{-1}$ due to general relativity, and removing his value of $+5024''\cdot93\ \mathrm{cy}^{-1}$ for general precession and replacing it by $+5026''\cdot74\ \mathrm{cy}^{-1}$ determined recently by Fricke (1967) from stellar kinematics. These steps are set out below:

Corrections to Newcomb's value for the motion of the Earth's perihelion

	$-10''\cdot45\ \mathrm{cy}^{-1}$	Newcomb's empirical term (*Elements*, p. 184)
	$+3''\cdot84\ \mathrm{cy}^{-1}$	General Relativity
	$-5024''\cdot93\ \mathrm{cy}^{-1}$	Precession in Newcomb's theory (*Elements*, p. 186)
	$+5026''\cdot74\ \mathrm{cy}^{-1}$	Recent determination by Fricke (1967)
$\Delta\dot{\pi}' =$	$-4''\cdot80\ \mathrm{cy}^{-1}$	Total

$\therefore\ e'\,\Delta\dot{\pi}' =$ $-0''{\cdot}080\ \text{cy}^{-1}$
$-0''{\cdot}010\ \text{cy}^{-1}$ Correction from revision of masses above

$\therefore\ e'\,\Delta\dot{\pi}' =$ $-0''{\cdot}090\ \text{cy}^{-1}$

We substitute this final correction for $e'\,\Delta\dot{\pi}'$, together with those for $\Delta\dot{e}'$ and $\Delta\dot{e}$ derived from the corrections to the masses, in the expressions for $\dot{V}$ and $\dot{W}$ (relations (4) and (5)):

$$+1{\cdot}49\ \Delta\dot{L} - 0{\cdot}49\ \Delta\dot{\pi} - 1{\cdot}08\ \Delta\dot{e} + 1{\cdot}19\ e'\,\Delta\dot{\pi}' + 1{\cdot}58\ \Delta\dot{e}' = +1''{\cdot}17 \pm 0''{\cdot}13$$

$$+0{\cdot}72\ \Delta\dot{L} + 0{\cdot}28\ \Delta\dot{\pi} + 0{\cdot}90\ \Delta\dot{e} - 1{\cdot}11\ e'\,\Delta\dot{\pi}' - 1{\cdot}62\ \Delta\dot{e}' = +0''{\cdot}76 \pm 0''{\cdot}12.$$

We have inserted solution 2 from Table III for $\dot{V}$ and $\dot{W}$. Hence,

$$+1{\cdot}49\ \Delta\dot{L} - 0{\cdot}49\ \Delta\dot{\pi} = +1''{\cdot}271 \pm 0''{\cdot}13$$

$$+0{\cdot}72\ \Delta\dot{L} + 0{\cdot}28\ \Delta\dot{\pi} = +0''{\cdot}661 \pm 0''{\cdot}12$$

assuming that all the uncertainty arises from $\Delta\dot{L}$ and $\Delta\dot{\pi}$. The solution is

$$\Delta\dot{L} = +0''{\cdot}88 \pm 0''{\cdot}10\ \text{cy}^{-1}$$

$$\Delta\dot{\pi} = +0''{\cdot}09 \pm 0''{\cdot}28\ \text{cy}^{-1}.$$

The correction, $\Delta\dot{\pi}$, is to be applied to the provisionally adopted value for the motion of Mercury's perihelion in Newcomb's theory in order to bring it into agreement with observation. We have for the epoch 1900·0:

Motion of perihelion in Newcomb's theory (*Elements*, p. 185)	$+5599''{\cdot}76$ cy^{-1}
Correction from this paper	$+0''{\cdot}09 \pm 0''{\cdot}3$ cy^{-1}
Revised observed motion of perihelion	$+5599''{\cdot}85 \pm 0''{\cdot}3$ cy^{-1}

The theoretical motion of the perihelion due to Newtonian gravitational attraction is given by the value in Newcomb's theory, less his empirical term. The longitude of perihelion is measured from the mean equinox of date, along the ecliptic to the ascending node, and then along the orbit. We revise his value for the motion of the perihelion in the plane of the orbit due to the changes in the planetary masses given above and substitute Fricke's value for the general precession. The contributions to the motion of the perihelion due to changes in the motions of the node and the plane of the ecliptic (*Elements*, p. 184) are negligible. We have for the epoch 1900·0:

Motion of perihelion in Newcomb's theory	$+5599''{\cdot}76$ cy^{-1}
Less Newcomb's empirical term (*Elements*, p. 184)	$-43''{\cdot}37$ cy^{-1}
Correction for change in masses (this paper)	$-0''{\cdot}22 \pm 0''{\cdot}01$ cy^{-1}
Remove value of precession in Newcomb's theory (*Elements*, p. 186)	$-5024''{\cdot}93$ cy^{-1}
Add revised value of precession (Fricke 1967)	$+5026''{\cdot}74 \pm 0''{\cdot}10$ cy^{-1}
Revised computed (Newtonian) motion of perihelion	$+5557''{\cdot}98 \pm 0''{\cdot}10$ cy^{-1}

∴ *Excess motion of perihelion of Mercury:*
Observed minus computed (*Newtonian*) = $+41''{\cdot}9 \pm 0''{\cdot}3$ cy^{-1}.

The value given here for the standard error does not include any allowance for the systematic errors; these are discussed in Section 5.3 and the final result is given in Section 5.4.

The correction to the motion of the node in Newcomb's theory is obtained from

$$\dot{N} = \sin i(\Delta\dot{\theta} - \Delta\dot{l}').$$

Since the value for the mean motion of the Sun in Newcomb's theory is treated as an absolute constant for the purpose of this analysis (see Section 4.2), we have

$$\Delta\dot{l}' = -2e'\,\Delta\dot{\pi}'\cos(L'-\pi') + 2\,\Delta\dot{e}'\sin(L'-\pi').$$

If we insert the values for $e'\Delta\dot{\pi}'$ and $\Delta\dot{e}'$ which we derived above from corrections to the planetary masses and the general precession we find $\Delta\dot{l}' = -0''{\cdot}10\ \mathrm{cy}^{-1}$ in May and $+0''{\cdot}10\ \mathrm{cy}^{-1}$ in November. The unknowns in latitude for the May and November transits were combined in the equation of condition on the assumption that $\Delta\dot{l}' \times \sin i$ would be much smaller than the uncertainty of $\dot{N}$. This is the case, so we neglect $\Delta\dot{l}'$ and find from solution 2 that

$$\Delta\dot{\theta} = -0''{\cdot}2 \pm 1''{\cdot}0\ \mathrm{cy}^{-1}.$$

This is the correction to be applied to Newcomb's theoretical value of $4266''{\cdot}75\ \mathrm{cy}^{-1}$ for the motion of the node in order that it should agree better with observation.

Motion of the node in Newcomb's theory (*Elements*, p. 185)	$+4266''{\cdot}75\ \mathrm{cy}^{-1}$
Correction from this paper	$-0''{\cdot}2 \pm 1''{\cdot}0\ \mathrm{cy}^{-1}$
Revised observed motion of the node	$+4266''{\cdot}5 \pm 1''{\cdot}0\ \mathrm{cy}^{-1}$

Newcomb's theoretical value of the motion of the node should be revised for the changes in the planetary masses and the general precession. We have already calculated the change with respect to the *fixed* ecliptic of 1900·0. The contribution due to the change in the motion of the ecliptic is calculated from the expression given by Newcomb (*Elements*, p. 183):

$$\sin i\,\Delta\dot{\theta} = -\Delta[\kappa''\cos i\sin(L''-\theta)]\ \mathrm{cy}^{-1}.$$

Inserting the values calculated above for $\Delta(\kappa''\sin L'')$ and $\Delta(\kappa''\cos L'')$, and taking $\theta = 47^\circ\,09'$, we find $\Delta\dot{\theta} = +0''{\cdot}28\ \mathrm{cy}^{-1}$. Bringing together the corrections to Newcomb's value, we have:

Motion of the node in Newcomb's theory (*Elements*, p. 185)	$+4266''{\cdot}75\ \mathrm{cy}^{-1}$
Correction with respect to the fixed ecliptic of 1900·0 (this paper)	$+0''{\cdot}11 \pm 0''{\cdot}01\ \mathrm{cy}^{-1}$
Correction for the motion of the ecliptic (this paper)	$+0''{\cdot}28 \pm 0''{\cdot}01\ \mathrm{cy}^{-1}$
Remove value of precession in Newcomb's theory (*Elements*, p. 186)	$-5024''{\cdot}93\ \mathrm{cy}^{-1}$
Add revised value of precession (Fricke 1967)	$+5026''{\cdot}74 \pm 0''{\cdot}10\ \mathrm{cy}^{-1}$
Revised computed (Newtonian) motion of the node	$+4268''{\cdot}9 \pm 0''{\cdot}1\ \mathrm{cy}^{-1}$

∴ *Excess motion of the node of Mercury:*
Observed minus computed (*Newtonian*) $= -2''{\cdot}4 \pm 1''{\cdot}0\ \mathrm{cy}^{-1}$.

This result does not include any allowance for systematic errors.

5.3 *Systematic errors*

The biases in timing contacts 2 and 3 will usually be in the opposite sense and, therefore, will add in the observed duration of a transit, but will subtract in the mean of the contacts taken together. For this reason, these biases should cause greater systematic errors in the deduced correction to latitude (N), than to longitude (V and W). However, the biases in duration will contribute opposite amounts to the solution for N depending on which hemisphere of the Sun the transit takes place; so, the resultant systematic corrections in N, and V and W, may be considered as being about equal in size. Since the coefficients of N in the observational equation (2) for November and May have opposite signs, the resultant systematic error due to biases in timing will enter the two solutions for N in the opposite sense. Thus, half of the difference between the two solutions will give an estimate of the systematic error in the solution for N, and hence V and W.

The separate solutions for the November and May transits, excluding observations which produced residuals lying outside the limits discussed in Section 4.1, are displayed in Table VI. (*cf.* Solution 2 in Table III.)

From the differences between the solutions for N_0 and $\dot{N}$ in Table VI, we estimate the systematic errors of our results for the various orbital parameters to be as follows:

Estimated systematic errors

ΔL_0	$\pm 0''\cdot 2$
$\Delta \pi_0$	$\pm 0''\cdot 4$
$\Delta \theta_0$	$\pm 2''$
$\Delta \dot{L}$	$\pm 0''\cdot 2$ cy^{-1}
$\Delta \dot{\pi}$	$\pm 0''\cdot 4$ cy^{-1}
$\Delta \dot{\theta}$	$\pm 2''$ cy^{-1}.

5.4 *Excess motion of Mercury's perihelion*

All the recent discussions of the optical data (Clemence 1943; Duncombe 1958; Wayman 1966), which find a value close to 43″ cy^{-1} for the non-Newtonian motion of the perihelion of Mercury, are based on the observed value deduced by Clemence (1943) from a comprehensive study of meridian-circle and transit observations. For the latter, he took the mean O–C times for each contact given by Williams (1939) for the transits of 1799–1927 and added data for the transit of 1940. Williams took his mean observed times of contact for each transit, except that of 1927 which he reduced himself, from the work of Newcomb (1882) and Innes (1925). Therefore, in his analysis, Clemence had a total of 25 observational equations relating the unknowns to the *mean* values of O–C for each observed contact of the transits from 1799 to 1940. For the transits before 1927 Clemence (via Williams) adopted the weights for the observational equations deduced by Innes from the internal agreement of the observations of each contact.

We find the following results for the correction to Newcomb's value of the motion of the perihelion from the work of Clemence (1943, p. 57; solving his equations in K and H, after substituting our values for $\Delta\dot{e}$, $e'\Delta\dot{\pi}'$ and $\Delta\dot{e}'$, and converting his probable errors to standard errors):

$$\Delta\dot{\pi} = +0''\cdot 8 \pm 0''\cdot 7 \text{ cy}^{-1} \quad \text{(Clemence; transits).}$$

TABLE VI

Separate solutions for the unknowns in equation (3) for the November and May transits

	Transits November (23) 1348 obs.	Transits May (10) 832 obs.
V_0 (″)	+0·14 ±0·11	—
$\dot{V}$ (″ cy^{-1})	+1·18 ±0·16	—
W_0 (″)	—	+0·51 ±0·05
$\dot{W}$ (″ cy^{-1})	—	+0·88 ±0·08
N_0 (″)	−0·72 ±0·11	−0·34 ±0·09
$\dot{N}$ (″ cy^{-1})	−0·05 ±0·18	+0·37 ±0·15
$\bar{R}$ (″)	+0·05 ±0·02	−0·07 ±0·02
k (″ cy^{-2})	−3·5 ±0·9	−2·9 ±1·4
Σ (residual)^2	6929	897
Variance	5·2	1·1

From his analysis of meridian–circle observations (Clemence 1943, p. 57, equation in II with $e' \Delta\dot{\pi}' = -0''{\cdot}085$ and $\Delta\dot{e}' = 0$), we find:

$$\Delta\dot{\pi} = +0''{\cdot}2 \pm 1''{\cdot}0 \text{ cy}^{-1} \quad \text{(Clemence; meridian–circle).}$$

Our result is:

$$\Delta\dot{\pi} = +0''{\cdot}1 \pm 0''{\cdot}5 \text{ cy}^{-1} \quad \text{(this paper).}$$

We have combined the standard error from the least-squares analysis with the systematic error estimated in Section 5.3. If Clemence's result from the transits is used in the computation of the non-Newtonian motion of the perihelion, we obtain $+42''{\cdot}6 \pm 0''{\cdot}7$ cy^{-1}: our solution gives $+41''{\cdot}9 \pm 0''{\cdot}5$ cy^{-1}. Clemence's result is not inconsistent with ours, but we believe that ours is to be preferred since we have used all the data in our analysis, rather than adopting mean times of contact, and we have doubled the time-range by including 16 more transits. Provided our estimate of the systematic error is realistic, our result would indicate that the relativistic component in the motion of Mercury's perihelion is slightly less than the value of $43''{\cdot}03$ cy^{-1} (Duncombe 1958) predicted by Einstein's general theory of relativity.

5.5 *General precession of the Earth's equator*

In order to derive the excess motions of the perihelion and node of Mercury's orbit we have to reduce our Earth-based observations to an inertial frame by removing the precession of the Earth's equator. In our analysis we have adopted the value of general precession derived by Fricke (1967) from stellar kinematics. Alternatively, we could have adopted the value of $+43''{\cdot}03$ cy^{-1} predicted by the theory of general relativity and regarded the general precession as the unknown to be determined from the motions of the perihelion and nodes. However, so long as there is some uncertainty about the size of the relativistic effect, and an uncertainty of about $\pm 2''$ cy^{-1} in the observed motion of the node, there is little possibility of deriving a reliable and independent value for the general precession from these data.

6. CONCLUSIONS

We have analysed about 2400 observations of the universal times of internal contact for the transits of Mercury in the period 1677–1973 in order to determine corrections to Newcomb's values for the orbital elements of Mercury. The distortions in the observations due to the non-uniformity of the universal time scale were removed by reducing the observations to an ephemeris time scale, which had been determined previously from observations of the motion of the Moon. It was recognized that this ephemeris time scale might itself depart from uniformity with the square of time due to a possible error in the adopted value of the tidal acceleration in the lunar ephemeris. So, in the observational equation of condition, we included an unknown in T^2, as well as constant and linear terms in T arising from corrections to the orbital elements. The solution of the observational equations by the method of least-squares lead to the following results:

$$\text{Tidal acceleration of the Moon} = -26'' \pm 2'' \text{ cy}^{-2}.$$

The uncertainty comprises a standard error of $\pm 0''.7$ and a contribution of about $\pm 2''$ for the total systematic uncertainty which was estimated from the spread of the results for different sub-sets of the data.

The corrections to Newcomb's values of the orbital elements of Mercury are:

Mean longitude	$+0''.3 \pm 0''.2 + (0''.9 \pm 0''.2)\,T$
Longitude of perihelion	$+0''.9 \pm 0''.5 + (0''.1 \pm 0''.5)\,T$
Longitude of node	$-5''.7 \pm 2''.1 - (0''.2 \pm 2''.2)\,T$
(Eccentricity	$-0''.59 \pm 0''.08$),

where T is centuries from the epoch 1900 January 0.5 UT. The uncertainties are standard errors combined with estimates of the systematic errors. In deriving these corrections, we adopted results from meridian–circle observations for the constant of eccentricity of Mercury's orbit and for the constants of eccentricity and longitude of the perihelion of the Earth's orbit. The values of the theoretical centennial variations of these elements were revised using recent determinations of the planetary masses and the general precession ($5026''.7$ cy^{-1}). The corrections given above to the centennial motions of the node and perihelion of Mercury's orbit lead to the following results when compared with Newtonian theory:

Excess motion of the node:	$-2''.4 \pm 2''.2$ cy^{-1}
Excess motion of the perihelion:	$+41''.9 \pm 0''.5$ cy^{-1}.

The correction to the motion of the node is not significant, but that to the perihelion does appear to be slightly less than that predicted by Einstein's general theory of relativity.

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