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## ON THE SYSTEM OF ASTRONOMICAL CONSTANTS

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1. *Introduction.* The national ephemerides published by several countries serve three distinct purposes. First, they constitute the basis of navigational almanacs and other extracted material, which are used by navigators, surveyors, engineers, other scientists, and the general public, as a means of accomplishing non-astronomical purposes. Second, they contain predictions of astronomical phenomena such as eclipses, occultations, and rising and setting of the moon, which are published in order to facilitate observations; the end sought may be either astronomical or non-astronomical. Third, they contain precise ephemerides which are intended to be compared with observations; analysis of the discrepancies between the ephemerides and the observations permits improvement of the constants of the theory on which the ephemerides are based, such as the masses of the planets and the elements of their orbits, and in addition has often indicated inadequacies in the theories.

The first two of these three purposes are, in general, of passing interest and of transient value to astronomers. It matters not what methods or what values of the constants are used in compiling the data, so long as the results are sufficiently accurate for the purposes at hand. The third purpose, on the contrary, is of first-class astronomical importance and for this purpose the data are of permanent value. If the discrepancies between theory and observation are to be correctly interpreted, it is necessary that the ephemerides truly represent the theories within amounts that are smaller than the errors of the observations; and furthermore, if the interpre-

tation is to be clear and unmistakable it is absolutely necessary that the theories be perfectly self-consistent. This last aim has never been completely attained, but during the present century we have been nearer to it than ever before. It used to be the fashion, for example, to calculate the perturbations of the earth by Venus and of Mars by Venus using two different masses of Venus; the resulting geocentric ephemeris of Mars involved both of these quantities, and it was therefore impossible to correct the assumed mass by comparing observations of Mars with the ephemeris. Such inconsistencies deprive precise ephemerides of most of their value, if not all of it; they render the ephemerides useful only for finding purposes, and nullify most of the not inconsiderable labor that has gone into their construction.

Fortunately the inconsistencies in the present official system of astronomical constants are not of this vicious kind, and with care the differences between observation and theory can now be correctly interpreted. The chief inconsistencies in the present system are:

*a.* The time used in the national ephemerides is the kind of time that satisfies the equations of celestial mechanics, often called Newtonian time, while the time used by astronomers in practice depends on the variable rotation of the earth. The use of one kind of time for the theory and another for the observations produces discrepancies between observation and theory which are proportional to the mean motions of the objects observed.

*b.* The adopted lengths of the sidereal year and the sidereal periods of all the planets are inconsistent with the adopted value of the general precession in longitude.

*c.* Empirical terms not resulting from gravitational theory are present in the ephemerides of the sun, moon, Mercury, Venus, and Mars. They were inserted solely for the purpose of making the ephemerides represent past observations, and they have failed to represent later observations. It is now known that the theory of relativity and the variable rotation of the earth obviate the necessity for such empirical terms.

*d.* The ratio of the mass of the earth to that of the moon used in the lunar tables is 81.53, that in the solar tables is 81.45, while yet a third value corresponds to the adopted constants of nutation and precession.

*e.* The adopted value of the solar parallax is  $8''.80$ , which is inconsistent with the adopted value of the constant of aberration  $20''.47$ . Both are inconsistent with the solar parallax used in the lunar tables,  $8''.80549$ , and with the mass of the earth used in the tables of the four inner planets, which corresponds to  $8''.79$ .

*f.* The parallactic inequality in the moon's longitude is inconsistent with the adopted value of the solar parallax.

*g.* The constant of the lunar parallax is inconsistent with the adopted values of the dimensions of the earth and the acceleration of gravity.

I limit myself to the inconsistencies that are susceptible of observation. It is true that there will always be inconsistencies between the adopted values of various constants in the  $n$ th decimal, unless their values are carried to  $n + 1$  decimals, but this is of no consequence to anyone. As an example of such discrepancies it may be mentioned that the masses of the planets printed in the tables of the eight principal planets are not identical in the end figures, but this is of no consequence because, with one exception, it is possible to choose for each mass a definite value that will represent its effects in all the tables to a fraction of the last decimal printed. The masses given later satisfy this requirement.

In addition to being self-consistent it is important that the values of the constants adopted in the official system should be reasonably near the true values. This requirement is much less stringent than the others. It is only required, in strictness, that the squares of the errors shall be negligible, but astronomers have generally been

influenced by the desire that the adopted value of a constant should be the best possible. This desire has occasionally led to changes in the official system of constants that were productive of more harm than good. The introduction of Newcomb's value of the precession, for example, has not lightened the labor of any astronomer, but on the contrary has caused much useless labor and difficulty for everyone who has tried to compare observations made before the change with those made afterward. I think that no one who has experienced these difficulties has felt himself recompensed by the knowledge that Newcomb's value was a little "better" than the one it superseded.

Notwithstanding the cogent reasons for leaving the present system unchanged I have thought it worthwhile to attempt to derive a system satisfying slightly different requirements from those laid down in previous works on the subject, which might be suitable for adoption if any change in the present system should be deemed necessary before the present lunar, solar, and planetary tables are superseded; this is the object of the present article. The requirements are:

*a.* The system is to be completely self-consistent.

*b.* The adopted value of each constant shall agree as nearly as possible with that given by observation.

*c.* The changes in the lunar, solar, and planetary tables shall be kept to the minimum consistent with *a* and *b*, that is, whenever a judicious consideration of the evidence leaves an element of arbitrary choice in the value of a constant, that value shall be chosen which entails the fewest corrections to the tables.

Some explanation of requirement *c* may be in order in view of the unfamiliarity of many persons with the details of construction of the tables. Inattention to it would speedily lead to a system that would require the tables to be completely reconstructed, which would be an onerous and exhausting task of several years duration, not sanely to be considered until the theories on which the tables are based can be improved.

It is well known that requirement *b* cannot be completely satisfied; the observed values of the constant of nutation and the constant of the lunar inequality cannot be reconciled and the system will necessarily be defective in this respect. This defect alone may be regarded as rendering the system unsatisfactory for practical

use, but the present work will nevertheless have value as a model of the procedure to be followed when the present system comes finally to be superseded.

Requirement *a* must be satisfied, whatever the consequences; otherwise the system is not entitled to be called a system, since it fails to accomplish its chief object.

In this work I make use of the classical article on the system of astronomical constants by W. de Sitter,<sup>1</sup> edited and completed after his death by Dirk Brouwer, to which the reader is referred for most of the theoretical relations and for most of the notation used here without further explanation. In that article is derived a truly consistent system, which, when the article was written, might fairly have been regarded as the best obtainable, with perhaps the single exception of the treatment of the measure of time. It seems advisable now, however, to revise the numerical values in several respects. The mass of Venus 1/404,000 is certainly too large; while the weight of evidence seems to indicate that Newcomb's mass 1/408,000 ought to be increased slightly, the evidence is not conclusive, and in view of the practical difficulty of introducing the consequent correction in the ephemerides of the four inner planets I conclude that we cannot do better for the present than to adhere to Newcomb's value. Somewhat the same arguments apply to the other planetary masses. Particularly in the case of the earth's mass, it is fortunate that Spencer Jones' value of the solar parallax corresponds almost exactly to Newcomb's value.

2. *The measure of time.* It is well established that the rotation of the earth is variable, which causes fluctuations in mean solar time in addition to the secular effects caused by the slowing down of the earth due to tidal friction. In consequence the observed positions of the sun and planets deviate from their positions as given by the solar and planetary theories, by amounts equal to their motions in a time-interval equal to the accumulated error in mean solar time. The same is true of the moon, but in addition the tidal slowing down of the earth is accompanied by a real diminution in the moon's angular mean motion, which is not included in Brown's lunar theory.

Another way of describing the situation is that the solar and planetary theories give the coordinates of the sun and planets as functions of Newtonian time, whereas the observed positions are functions of variable time; hence the theoretical coordinates differ from the observed.

This inconsistency cannot be allowed to persist; it defeats the principal purpose for which tables of the sun, moon, and planets are constructed, which is to predict their positions. The inconsistency can conceivably be removed in either of two ways: (*a*) by introducing into the theories the non-Newtonian time unit hitherto used in practice, or (*b*) by applying corrections to our practical determinations of time in order to make it Newtonian.

In the present state of astronomy the first of these alternatives is entirely impracticable, because the rate of rotation of the earth (hitherto used for the practical measure of time) cannot be predicted in advance but must be determined at any epoch by observation.

The second alternative is the one that must be adopted, and the manner of applying the corrections must be considered. At first thought it might seem logical actually to apply the corrections to the indications of all the clocks on the earth, and thus to introduce Newtonian time into civil life. This would necessitate extrapolating the rate of rotation of the earth a few years into the future, in a way analogous to the extrapolations of clock corrections now made a few days into the future by all the national time services. The chief objection to this way of proceeding is that the clocks on the earth would depart further and further from mean solar time, the difference amounting to several hours in two thousand years, so that in A.D. 4000, for example, the sun might transit the meridian at 3 p.m. by the clock. It seems doubtful if such a discrepancy would be tolerated indefinitely; in spite of the fortitude with which citizens endure the spectacle of meridian transit of the sun at 1 p.m. or 2 p.m. in the summer, when clocks in many regions are advanced one or two hours arbitrarily, they will probably not tolerate a meridian transit at, say, 9 p.m.

It therefore seems logical to continue the use of mean solar time (or time differing from it by a constant arbitrary amount) for civil purposes, and to introduce Newtonian time for the convenience of astronomers and other scientists only, which can easily be done in the following way. Astronomers will continue to set their clocks to the mean solar time of the Greenwich meridian (Universal time), and to publish their observations on this standard of reference. Whenever an observation is to be compared with a theoretical position, the time of the observation will first be corrected to Newtonian time. The corrections to

Universal time needed to reduce it to Newtonian time can be published in the national ephemerides.

This method of bringing observations into agreement with theory has already been used to some extent. It is greatly facilitated by the work of Spencer Jones.<sup>2</sup> He has determined the correction to the sun's tabular mean longitude given by Newcomb's tables to be

$$\Delta L_{\odot} = +1''.00 + 2''.97 T + 1''.23 T^2 + 0.0748 B,$$

when the times of observation are in Universal time, not corrected to Newtonian time. In this expression  $T$  is the time in Julian centuries of 36525 mean solar days counted from 1900 January 0, Greenwich Mean Noon (Jan. 0.5 U.T.), and  $B$  is approximately the observed correction to the moon's tabular mean longitude, after certain corrections have been applied to the lunar tables, the times of observation again being in Universal time.

The sun's tropical mean longitude increases at the rate of 1 second of arc in 24,349 seconds of time, whence the correction to mean solar time required to transform it to Newtonian time is

$$\Delta t = +24^s.349 + 72^s.3165 T + 29^s.949 T^2 + 1.821 B.$$

Of these four terms only the last two are strictly needed in order to obtain a Newtonian measure of time, that is, a measure the unit of which bears a fixed ratio to the sidereal year, after allowing for the known gravitational acceleration of the earth's mean sidereal motion due to the action of the other planets. I leave aside the philosophical question whether the sidereal year so corrected is truly constant. So far as a Newtonian measure of time is concerned the first two terms remain arbitrary. It is desirable, however, to bring the Newtonian day into some sort of resemblance to the mean solar day as it exists in our time, by giving suitable values to the first two terms, which determine the epoch at which the Newtonian day commences, and its length. The values of these terms given above have approximately the effect of making the Newtonian day coincide with the mean solar day at 1900, both in epoch and in duration. Speaking rigorously, the coefficient of  $T$  is intended to fix the length of the Newtonian day to agree with the length of the mean solar day at 1900 January 0, the mean solar day being that given by the formula in Newcomb's tables of the sun; and the constant term fixes the coincidence of Universal time and Newtonian time at an epoch near 1900 January 0, at which the sum of the four

terms vanishes, mean solar time being determined by Newcomb's formula.

Spencer Jones finds that the correction required to the moon's tabular mean longitude, as given by Brown's tables, is

$$\Delta L_{\zeta} = +4''.65 + 12''.96 T + 5''.22 T^2 + B$$

— Brown's empirical term,

the actual mean longitude being determined by observations referred to Universal time. It is important to notice, however, that this expression ought not to be used as a definition of  $B$  if  $B$  exceeds a few seconds of arc. In the interval  $\Delta t$  not only the moon's mean longitude but all the other arguments of the lunar tables are changing, and all of these changes ought to be taken into account. The term  $1.821 B$  is, rigorously, that portion of the correction to mean solar time that is attributed to the irregular fluctuations in the rate of rotation of the earth.

In the time interval  $\Delta t$  the moon's mean longitude increases by

$$\Delta L_t = +13''.37 + 39''.71 T + 16''.44 T^2 + B.$$

Hence, when the observations of the moon are referred to Newtonian time the correction required to Brown's tables is

$$\Delta L_{\zeta} - \Delta L_t = -8''.72 - 26''.75 T - 11''.22 T^2$$

— Brown's empirical term,

which has the effect of introducing into the lunar theory the same unit of Newtonian time as is used for the theories of the sun and planets.

This correction ought to be introduced into the lunar ephemeris at the time when the official system of astronomical constants is revised. Thereafter  $B$  is to be determined as follows. The moon's observed right ascension and declination, referred to Universal time, will not agree with the tabular values. Each observation of the moon will furnish a value of  $\Delta t$  determined by the condition that the observed right ascension and declination must be brought into agreement with the tabular values. These values of  $\Delta t$ , averaged over any desirable number of weeks or months, are to be regarded as definitive, and  $B$  may be determined, if desired, by the formula

$$1.821 B = \Delta t - 24^s.349 - 72^s.3165 T - 29^s.949 T^2.$$

In order to clarify the ideas I mention a few of the more important consequences arising from the two different measures of time.

A. Consequences of the unavoidable use of Newtonian time in gravitational theories and the

use of Universal time as the reference for observations, as has been done heretofore.

*a.* The sun and principal planets (except Pluto) depart from their ephemerides by continually increasing amounts. During a single century the effects are noticeable chiefly in the mean longitudes, but over a long period of time they become appreciable in the longitudes of the perihelia and nodes. In the case of the sun, the effect on the mean longitude is given approximately by  $\Delta L_{\odot}$ . For the major planets the effects are approximately given by the ratio of the mean motions to the earth's mean motion. The same is true for any other celestial body (including those outside the solar system) for which the elements of the orbit have been determined by observations extending throughout the nineteenth century. For bodies having elements differently determined the effects are different; they can be calculated from the relation between Newtonian time and Universal time over the period covered by the observations.

*b.* The moon departs from its ephemeris by a continually increasing amount. The effect on the mean longitude is given approximately by  $\Delta L_{\zeta}$ .

*c.* A clock keeping Universal time suffers a continuous deceleration. Its average value over the past 2000 years is 29 seconds per century per century. At present it is somewhat less. In addition the clock undergoes irregular changes in rate, which cannot be predicted in advance, but which in the past have been such as to cause errors amounting to more than 30 seconds.

B. Consequences of the use of Newtonian time, as proposed above, both in gravitational theories and for observations.

*a.* The sun, moon, planets, and all other bodies continuously agree with their ephemerides, with the possible exception of small discrepancies caused by defects in the gravitational theories of their motions.

*b.* A clock keeping Newtonian time would run with a rate which, so far as our present state of knowledge is concerned, is constant. Our practical determinations of Newtonian time will not be perfect, however, but will be affected by very small errors caused by defects in the lunar theory. These errors cannot be detected by any observations except those of the moon itself. They are entirely negligible in the present state of astronomy, being much smaller than those caused by the practice of determining time by referring the hour angles of celestial bodies to the instantane-

ous meridian, which is still being done except at the Royal Greenwich Observatory.

*c.* As the rate of rotation of the earth continues to diminish due to tidal friction, the sun will come to the meridian later and later by Newtonian time. By A.D. 3000 the accumulated effect will amount to something like half an hour, and the length of the mean solar day will have increased by more than 0.1 Newtonian second.

When the national ephemerides begin to publish the corrections needed to reduce Universal time to Newtonian time, it will be desirable to change the designations of time printed at the top of various pages of ephemerides from "Universal time" to "Newtonian time." This change will not be the result of any change in the ephemerides themselves, but will be simply a recognition of a situation that has always existed: namely, that the unit of time used in gravitational theories is a Newtonian unit. It may be expected, on the contrary, that astronomical data intended for the use of navigators and surveyors will be altered from time to time, as has been done in the past, and that the designation "Universal time" will continue to be the appropriate one to use in such publications.

3. *Adopted values of astronomical constants.* For the reciprocals of the planetary masses (including atmospheres and satellite systems) I adopt

Mercury	6 000 000
Venus	408 000
Earth + moon	329 406
Mars	3 093 500
Ceres	2 500 000 000
Jupiter	1 047.355
Saturn	3 501.6
Uranus	22 869
Neptune	19 314
Pluto	360 000

These masses are completely consistent with the current tables of the sun and principal planets, with the exception of Neptune, Ceres, and Pluto. They are also consistent with the lunar tables, with the exception of the earth + moon, which corresponds to a solar parallax of  $8''.79$ , whereas the value in the lunar tables is  $8''.80549$ . The mass of Neptune is that used in the tables of Uranus; its value in the other tables is  $1/19\,700$ . The masses of Ceres and Pluto are neglected in all the tables; they come into the system of constants only through their effect on the motion of the ecliptic. The mass of Ceres is

a very rough approximation, showing merely the order of magnitude. The mass of Pluto is virtually that resulting from the work of Brouwer and L. R. Wylie.<sup>3</sup>

For the general precession in longitude I take the value obtained by Oort<sup>4</sup> referred to the star-system of the FK3. This gives  $p = 5026.65$  seconds of arc per tropical century at 1900.

For the observed value of the constant of nutation there are four important results:

Newcomb, <sup>5</sup> from numerous determinations,	9".210
Przybyllok, <sup>6</sup> from results of the International Latitude Service,	9.207
Spencer Jones, <sup>7</sup> from observations with the Cookson telescope,	9.2066
Morgan, <sup>8</sup> from circumpolar observations at Washington,	9.206

The remarkable accordance of these determinations renders any discussion of the weights unnecessary for the present purpose. The general mean is 9".207.

For the observed value of the lunar inequality the following determinations deserve notice. The numbers represent the coefficient of the principal lunar inequality in the sun's longitude.

Newcomb, <sup>9</sup> sun	6".485 ± 0".012
Gill, <sup>10</sup> Victoria	6.443 ± 0.007
Hinks, <sup>11</sup> Eros	6.461 ± 0.002
Morgan and Scott, <sup>12</sup> sun	6.479 ± 0.010
Spencer Jones, <sup>13</sup> Eros, slightly revised by Jeffreys <sup>14</sup>	6.467 ± 0.001

The discordance among these determinations renders it almost certain that one or more of them are affected by systematic errors, and it is difficult to decide what weights ought to be assigned. For the moment I take for the general mean  $6".464 \pm 0".003$ , without regarding it as final.

The impossibility of reconciling this result with that for the constant of nutation may be shown by deriving the mass of the moon from the observed values of the precession and nutation, and then the lunar inequality from the mass of the moon. The result is 6".42, which shows clearly that either (a) systematic errors are present in one or both of the observational values, or (b) the theoretical relations are defective.

The theoretical relations involve the solar parallax, the lunar parallax, the lunar theory, and the theory of rotation of the earth. Both the theoretical relations and the observations

have been, and still are, the subjects of intensive study by a number of capable astronomers, and it is useless to speculate on the outcome. In the meantime it is necessary to accept the existing theory, and it is advisable to adopt whatever expedient will best facilitate further research on the subject, recognizing that whatever is done now, a change may be necessary later. It seems to me that the needs of astronomers will be best served by adhering to the present value of the constant of nutation, and accordingly I adopt 9".210.

The velocity of light is known with such accuracy that no admissible change will have appreciable astronomical consequences. I adopt the result of Dorsey's<sup>15</sup> exhaustive discussion of all determinations, 299 773 km/sec.

For the mean obliquity of the ecliptic at 1900 January 0, Greenwich Mean Noon, I take Newcomb's value,  $23^{\circ}27'8".26$ .

For the solar parallax I take the result of Spencer Jones' investigation,<sup>16</sup> 8".790.

4. *Geodetic constants.* By geodetic constants I mean those that specify the size, shape, and internal constitution of the ideal body that is of necessity substituted for the actual earth in astronomical computations. In the present official system of astronomical constants this body is an ellipsoid of revolution, the surface of which is completely specified by two constants, for which we may choose either the two principal axes, or one axis and the oblateness. In de Sitter's system it is a figure of revolution, specified by the two principal axes and two small constants,  $\kappa$  and  $\lambda_1$ , that describe the internal constitution. The figure is not an exact ellipsoid; in mid-latitudes it is depressed about 3 meters below the ellipsoid having the same principal axes.

The simple ellipsoid no longer suffices for astronomical purposes because the theoretical relation between the oblateness and the constant of precession is not sufficiently rigorous. But de Sitter's way of treating the matter is not the only possible one. It is equally feasible to use an exact ellipsoid with axes slightly different from de Sitter's, with the addition of a small zonal harmonic of the fourth degree in somewhat the manner described by Jeffreys<sup>17</sup> and by Lambert.<sup>18</sup> So far as the strict requirements of astronomy are concerned, I think it is indifferent which of these two courses is adopted; either will meet all foreseeable needs for the next hundred years. In the circumstances astronomers might well follow the lead of geodesists. The present tendency

TABLE I. SECULAR MOTION OF THE ECLIPTIC

	$\frac{d}{dt} (\pi \sin \Pi)$			$\frac{d}{dt} (\pi \cos \Pi)$		
	1600	1850	2100	1600	1850	2100
Mercury	+0".3088	+0".3138	+0".3175	- 0".2650	- 0".2625	- 0".2600
Venus	+6.8233	+7.4483	+8.0714	-28.6126	-28.4709	-28.3232
Mars	+0.6170	+0.6340	+0.6510	- 0.7350	- 0.7190	- 0.7030
Ceres	+0.0001	+0.0001	+0.0001	- 0.0004	- 0.0004	- 0.0004
Jupiter	-2.8054	-2.5123	-2.2251	-16.1781	-16.0550	-15.9270
Saturn	-0.5740	-0.5420	-0.5100	- 1.3100	- 1.3180	- 1.3250
Uranus	+0.0020	+0.0020	+0.0030	- 0.0080	- 0.0080	- 0.0080
Neptune	-0.0040	-0.0040	-0.0040	- 0.0040	- 0.0040	- 0.0040
Pluto	-0.0004	-0.0004	-0.0004	- 0.0012	- 0.0012	- 0.0012
Total (Julian Century)	+4.3674	+5.3395	+6.3035	-47.1143	-46.8390	-46.5518
Total (Tropical Century)	+4.3673	+5.3394	+6.3034	-47.1133	-46.8380	-46.5508

seems to favor the second course, but the matter cannot yet be said to have been settled definitely. This is another reason why revision of the present system of astronomical constants might well be deferred. In the meantime for convenience I adopt the theory of de Sitter, without intending to prejudice future discussions.

For the equatorial radius I take the conventional value,  $b = 6\,378\,388$  meters, and for the two small constants,  $\kappa = 0.000\,000\,50$ ,  $\lambda_1 = 0.000\,40$ . The oblateness will be derived by de Sitter's theory from the observed value of the precession.

For the acceleration of gravity on the equator I take the conventional value diminished by 15 milligals,  $g_0 = 979.034$  cm/sec<sup>2</sup>.

The uncertainties in the values of these four constants are so small that the astronomical effects are entirely negligible, with the exception of  $b$  and possibly  $g_0$ . The value of  $b$  is in doubt by a quantity of the order of 100 meters, which would alter the lunar parallax by 0".018. A change of 4 milligals in  $g_0$  would alter the lunar parallax by 0".005.

5. *Motion of the ecliptic.* For the secular mo-

tion of the plane of the earth's orbit under the influence of the other planets I utilize the developments of Newcomb,<sup>19</sup> reduced to the masses of Section 3. It seems desirable to add the effect of Pluto, and of Ceres, the most massive of the minor planets, more in order to show the order of magnitude of the effects than because of their importance. These I have obtained by a method described by Hill,<sup>20</sup> omitting from the disturbing function all terms higher than the second order with respect to eccentricities and inclinations. The results are given in Table I for three epochs. The fourth decimal is significant only for Ceres and Pluto. It is necessary to observe that all the quantities in Table I are accurate only to the first order of the masses. The terms of higher order are entirely unknown; they are probably unimportant but they may not be negligible, and they ought to be investigated.

From the quantities in Table I are obtained the following developments in powers of the time,  $T_1$  being reckoned in tropical centuries from 1850.0, in which the end figure of each coefficient is fictitious.

$$\begin{aligned}\pi \sin \Pi &= + 5".3394 T_1 + 0".19361 T_1^2 - 0".000216 T_1^3, \\ \pi \cos \Pi &= -46".8380 T_1 + 0".05625 T_1^2 + 0".000317 T_1^3.\end{aligned}$$

From these expressions and  $p = 5026".650$  I obtain, making use of developments similar to Andoyer's,<sup>21</sup>  $T$  being reckoned in tropical centuries from 1900.0,

$$\begin{aligned}\pi \sin \Pi &= + 4".962 T + 0".1940 T^2 - 0".00022 T^3, \\ \pi \cos \Pi &= -46".845 T + 0".0543 T^2 + 0".00032 T^3, \\ \Theta &= 23^\circ 27' 8".26 - 46".845 T - 0".0043 T^2 + 0".00180 T^3, \\ \Theta_1 &= 23^\circ 27' 8".26 + 0".0606 T^2 - 0".00774 T^3, \\ p_1 &= 5038".089 + 0".4930 T - 0".00008 T^2, \\ p &= 5026".650 + 2".2248 T - 0".00061 T^2, \\ \lambda &= 12".469 - 1".8889 T + 0".00076 T^2, \\ m &= 4609".432 + 2".7966 T - 0".00087 T^2, \\ n &= 2005".086 - 0".8535 T - 0".00028 T^2, \\ d\pi/dT &= 47".108 - 0".0709 T + 0".00057 T^2, \\ \Pi &= 173^\circ 57' 11" + 3288".6 T - 0".57 T^2.\end{aligned}$$

In these expressions  $\Theta$  is the angle between the instantaneous mean equator and the instantaneous ecliptic,  $\Theta_1$  that between the instantaneous mean equator and the ecliptic of 1900.0,  $p_1$  is the lunisolar precession diminished by the geodesic precession,  $p$  is the general precession in longitude,  $\lambda$  is the planetary precession,  $m$  is the general precession in right ascension,  $n$  is the general precession in declination,  $d\pi/dT$  is the speed of rotation of the ecliptic, and  $\Pi$  is the longitude of the ascending node of the moving ecliptic on the fixed ecliptic of 1900.0, measured from the mean equinox of date. All of the precessions are instantaneous rates of motion per tropical century.

6. *Secular variations.* From the measure of time adopted in Section 2 it follows that the correction to the sun's tabular mean longitude is zero, and that the correction to the moon's tabular mean longitude is

$$-8''.72 - 26''.75 T - 11''.22 T^2 - \text{empirical term.}$$

We have the following terms in the sun's centennial mean motion: in the sidereal mean motion, due to secular perturbations by the planets,<sup>22</sup>  $-0''.0403 T$ ; in the general precession,  $+2''.2248 T$ ; and hence in the tropical mean motion,  $+2''.1845 T$ . For the moon we have: secular perturbations by the planets,<sup>23</sup>  $+12''.048 T$ ; secular perturbations from slowing down of the earth,  $-22''.44 T$ ; hence in the sidereal mean motion,  $-10''.392 T$ ; and in the tropical mean motion,  $-8''.1672 T$ , in which the last two figures are fictitious.

7. *Mean motions.* The sun's geometric mean longitude, referred to the mean equinox of date, is  $279^\circ 41' 48''.04 + 129\,602\,768''.130 T + 1''.092 T^2$ , where  $T$  is now reckoned in Julian centuries of 36525 Newtonian days. Subtracting the general precession in a Julian century gives for the sidereal mean motion in a Julian century of 36525 Newtonian days,

$$129\,597\,741''.373 - 0''.040 T.$$

The sidereal mean motion in a Newtonian day is

$$3548''.192\,7823 - 0''.000\,0011 T.$$

The number of Newtonian days in a sidereal year is

$$365^d 256\,365\,56 + 0^d 000\,000\,11 T.$$

The number of Newtonian days in a tropical year is

$$365^d 242\,198\,78 - 0^d 000\,006\,16 T.$$

For the moon the tropical mean motion in a Julian century is

$$1\,732\,564\,379''.31 - 8''.167 T + 0''.0204 T^2,$$

whence it follows that the sidereal motion is

$$1\,732\,559\,352''.55 - 10''.392 T + 0''.0210 T^2$$

and the sidereal mean daily motion

$$47\,434''.889\,8713 - 0''.000\,2845 T + 0''.000\,000\,57 T^2.$$

The right ascension of the fictitious mean sun is

$$18^h 38^m 45^s 836 + 8\,640\,184^s 542 T + 0^s 0932 T^2.$$

The sun moves 1 second of arc in tropical longitude in 24.349 48 seconds of Newtonian time; and the moon in 1.821 439. The Besselian year commences when the right ascension of the fictitious mean sun is  $18^h 40^m$ . For 1900 this is January 0.813 516 Newtonian time. For any other year it is

$$0^d 813\,516 + 0^d 242\,198\,78 (x - 1900) - 0^d 000\,308 T^2 - n,$$

where  $n$  is the number of leap years between  $x$  and 1900 (not counting  $x$  itself).

The sidereal motion of the fictitious mean sun in a Newtonian day is  $3548''.204\,2053$ . The rotation of the earth in a Newtonian day is variable; its average value during the past 2000 years has been

$$1\,299\,548''.204\,2053 - 0''.0246 T$$

and it is accomplished in precisely 86 400 Newtonian seconds.

The rotation of the earth between two consecutive passages of the fictitious mean sun over the mean meridian of any place is  $1\,299\,548''.204\,2053$ , and it is accomplished in  $86\,400 + 0.00164 T$  seconds of Newtonian time, on the average.

A Newtonian day is defined as the interval during which the right ascension of the fictitious mean sun, measured from a fixed equinox, increases by  $3548''.204\,2053$ . A mean solar day is the interval between two successive transits of the fictitious mean sun over the meridian of any place.

8. *Fundamental and derived constants.* The fundamental constants  $R_1$  and  $g_1$  are determined by trial, so that the theoretical relations will repro-



TABLE II. FUNDAMENTAL AND DERIVED CONSTANTS

mean radius of the earth, meters,	$R_1 = 637\ 1248$
acceleration of gravity in mean latitude, cm/sec <sup>2</sup> ,	$g_1 = 979.749$
dynamical flattening, $(C - A)/C$ ,	$H = 0.003\ 286\ 885$
constants depending on the inner constitution of the earth,	$\begin{cases} \kappa = 0.000\ 000\ 50 \\ \lambda_1 = 0.000\ 40 \end{cases}$
solar parallax,	$\pi_{\odot} = 8''.790$
velocity of light, km/sec,	$c = 299\ 773$
reciprocal of moon's mass,	$\mu^{-1} = 81.79$
$\omega^2 R_1^3 / f M_1$ ,	$\rho_1 = 0.003\ 450\ 00$
$\frac{3}{2} C / M_1 b^2$ ,	$q = 0.501\ 18$
$\frac{3}{2} (C - A) / M_1 b^2$ ,	$J = 0.001\ 647\ 32$
reciprocal of oblateness,	$\epsilon^{-1} = 296.202$
equatorial radius, meters,	$b = 637\ 8388$
acceleration of gravity on the equator, cm/sec <sup>2</sup> ,	$g_0 = 978.034$
coefficient of $\sin^2 \varphi$ in gravity,	$\beta = 0.005\ 280\ 01$
coefficient of $\sin^2 2\varphi$ in gravity,	$\gamma = -0.000\ 007\ 35$
astronomical unit, km,	$149\ 670\ 000$
constant of aberration,	$k = 20''.507$
light-time,	$\tau = 499^s 291$ or $0^d 005\ 778\ 83$
mass-ratio, sun/(earth + moon),	$m^{-1} = 329\ 406$
constant of sine of lunar parallax,	$\pi' \zeta = 3422''.599$
parallactic inequality,	$P = 124''.941$
constant of lunar inequality,	$L = 6''.398$
lunar inequality in sun's longitude,	$L_s = 6''.427$
constant of precession,	$P = 5493''.847 - 0''.0036 T$
constant of nutation,	$N = 9''.210 + 0''.0009 T$
lunisolar precession,	$p_0 = 5040''.010 + 0''.4930 T$
geodesic precession,	$p_{\theta} = 1''.921$

duce the adopted values of  $b$  and  $g_0$ . The constants  $H$  and  $\mu^{-1}$  are determined from the adopted constants of precession and nutation. Then all the other constants are derived by the relations of de Sitter and Brouwer, noting that the coefficient of  $z$ , as used in their expressions for  $P$  and  $p_0$ , becomes  $-0.6741$  for  $\mu^{-1} = 81.79$ . The results are given in Table II.

#### 9. Precepts for introducing the new system.

a. Positions of stars. The proper motions of the stars given in fundamental catalogs such as the FK3, the GC, the NZC, and all others that have adopted Newcomb's precession, require correction. This correction is equal and opposite in sign to the correction to Newcomb's precession. The correction to the centennial proper motion in right ascension is

$$-0''.93 - 0''.41 \sin \alpha \tan \delta$$

or

$$-0^s 062 - 0^s 027 \sin \alpha \tan \delta,$$

and that to the centennial proper motions in declination is

$$-0''.41 \cos \alpha$$

where  $\alpha$  and  $\delta$  are the right ascension and declination of the star.

The *variations* in right ascension and declination published in these catalogs do not require any correction. Hence the procedure heretofore used in reducing positions of stars in these catalogs from 1950.0 to the beginning of any other year does not require any modification.

In photographic work it has often been the practice to reduce the star places of the astrographic catalog or of the AG catalog to the beginning of the current year or to 1950.0 without proper motion. Such reductions, and all ephemerides depending on them, are incorrect, as are the elements of the orbit of any object that depend on observations so reduced. The error amounts to less than a second of arc in the position of an object, however, and in practice it will usually be neglected; whenever the orbit is corrected by comparison with observations reduced on the new system the error previously arising from an incorrect precession will automatically be eliminated, but the systematic effects of solar motion and galactic rotation will still be present.

In reducing star positions from the beginning of the current year to apparent place or vice versa the new values of the constant of precession and the constant of aberration must be used.

*b.* Solar ephemeris. The correction to the sun's mean longitude,  $+0''.003 T^2$ , and to the right ascension of the fictitious mean sun,  $+0''.0003 T^2$ , being smaller than the unit of calculation, should be neglected. The correction to the centennial sidereal mean motion,  $-1''.83$ , has no effect on the calculation of ephemerides. The correction to the mean obliquity,  $+0''.0016 T^2 - 0''.00001 T^3$ , should be neglected. The tabular centennial motion of the perihelion should receive the following corrections:

for correction of the general precession,	+ 1''.83,
for removal of empirical term,	- 10.45,
for relativity effect,	+ 3.84,

total correction, - 4''.78.

Hence the mean anomaly should be corrected by  $+4''.78 T$ . This correction may be most conveniently introduced by adding  $+0''.0013 T$  to argument M of the solar tables.

To introduce the new value of the lunar inequality by correcting the coefficients given on page 18 of the solar tables would be somewhat laborious and the result would be affected by end-figure errors. It is better and easier to proceed rigorously, by calculating the complete value of the corrected perturbations for every day in the following manner. Let  $\pi_1$ ,  $\lambda_1$ ,  $\beta_1$  be the horizontal parallax, longitude, and latitude of the moon at any instant, and let  $\pi$ ,  $\lambda$ ,  $\beta$  be the corresponding quantities for the sun,  $\pi$  being obtained by  $8''.790/r$ , where  $r$  is the earth's radius vector. Then the corrections to  $\lambda$ ,  $\log r$ , and  $\beta$  are to be calculated by

$$\Delta\lambda = 2491'' \frac{\pi}{\pi_1} \cos \beta_1 \sin (\lambda_1 - \lambda),$$

$$\Delta \log r = 524600 \frac{\pi}{\pi_1} \cos \beta_1 \cos (\lambda_1 - \lambda),$$

$$\Delta\beta = 2491'' \frac{\pi}{\pi_1} \sin \beta_1,$$

where  $\Delta \log r$  is in units of the eighth decimal. The values of  $\Delta\lambda$  and  $\Delta \log r$  are to be increased by  $8''.00$  and  $1600$ , respectively, and the results used instead of those of tables 13-15, 23-25, and 30-31. The discrepancies between these precepts

and those on page 18 of the solar tables are due to revision of the mass of the moon, and to typographical errors.

The values of the precession, nutation, and aberration, given in the solar tables, have not been used since about 1917, nor will they be in the future.

*c.* Ephemeris of Mercury. The only corrections required are those arising from the motion of the perihelion and node. The tabular centennial motion of the perihelion requires the following corrections:

for correction of the general precession,	+ 1''.83,
for removal of empirical term,	- 43.37,
for relativity effect,	+ 43.03,
total correction,	+ 1''.49.

The correction to the mean anomaly is  $-1''.49 T$ , which may be introduced by applying  $-0''.000101 T$  to argument H of the Tables of Mercury.

The longitude of the node requires the correction  $+1''.83 T$ , which should be applied to  $\theta$ .

*d.* Ephemeris of Venus. The same sort of corrections are required as for Mercury. The correction to the mean anomaly is  $+6''.52 T$ , which gives for the correction to argument K,  $+0''.0011 T$ . The correction to  $\theta$  is  $+1''.83 T$ .

*e.* Ephemeris of Mars. The present ephemeris of Mars contains several empirical terms, and in addition, the theory itself is defective in important respects. Pending revision of the theory it does not seem worthwhile to modify the methods of calculating the ephemeris heretofore used.

*f.* Ephemerides of Jupiter and Saturn. From an examination of Hill's theory I conclude that the general precession used therein corresponds to  $5026''.894$  in a Julian century at 1900. The value in the present article is  $5026''.757$ . The source of Hill's value is unknown to me. The error of Hill's value produces an error in the longitude of Jupiter and Saturn of the order of  $0''.01 T$ . This error is probably much smaller than other errors in the theory, and it should be neglected.

*g.* The errors in the ephemerides of Uranus and Neptune arising from inconsistencies in the astronomical constants are so much smaller than other errors present that it is not worth while to correct them until the theories are revised.

*h.* The lunar ephemeris. The following additions to the arguments  $L$  and  $\tilde{\omega}'$ , and also to the

mean longitude, are to be introduced by the precepts on pages 138 and 139 of the lunar tables:

addition to  $L$  and the mean longitude,  
 $-8''.72 - 26''.75 T - 11''.22 T^2$ ,  
 addition to  $\bar{\omega}'$ ,  
 $-4''.78 T$ .

The new value of the constant of the sine of the parallax is introduced by the precept on page 140 of the tables, that is, by multiplying the parallax deduced from the tables by  $3422.599/3422.5400 = 1 + 0.00001724$ .

The parallactic terms cannot be corrected by the precepts on page 140, which are erroneous; Table 47, Section III is not affected by any change in  $\alpha_1$ . The necessary corrections may be found by multiplying all coefficients having  $\alpha_1$  for the principal characteristic by  $-0.001701$ . The corrections to four terms in the longitude and to one in the sine of the parallax are large enough to be appreciable. The corrections should be introduced as follows:

to $\Sigma_2$ the correction	$+0''.213 \sin D$ ,
to $\Sigma_1$ the correction	$+0''.014 \sin (l + D)$ $-0''.032 \sin (l - D)$ $-0''.031 \sin (l' + D)$ ,
to $\Sigma_9$ the correction	$+0''.0017 \cos D$ .

These corrections should be calculated at half-day intervals and the results rounded to  $0''.01$  for the longitude and  $0''.001$  for the parallax.

The corrections due to the new value of the oblateness may be introduced by the precept on page 140 of the lunar tables. It does not seem worth while in addition to correct the tabular motions of the perigee and node to make them conform to the new value of the oblateness. The theoretical motions are obtained by adding together about 20 contributions from different sources. Even if the separate contributions were all correct in the last figure, which is almost certainly not the case, the probable errors of the

total theoretical motions would be about  $1''.0$  per century. It seems desirable therefore to regard the tabular motions as empirical values, not to be corrected at present.

The empirical term should be removed by substituting for Table P24, Section VI, the constant 1100, and for Tables P27, P30, P33, the respective constants  $0^\circ 1543$ ,  $0^\circ 147$ ,  $0^\circ 181$ .

10. *Acknowledgments*. I am much indebted to W. D. Lambert for helpful advice on the geodetic constants, and to Dirk Brouwer and E. W. Woolard for critical review of the entire work with many valuable suggestions.

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1. *B. A. N.* 8, 213, 1938. The following corrections should be made. In the middle of page 215 insert the number (3) beside the formulae. On page 222 in formulae (46) and in the corresponding footnote, for 0.0172020985 read 0.01720209895. On page 225 in formulae (63), for 5093.376 read 5039.376. These errors are typographical only, and do not affect anything else in the article.
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