Cyril Potter, Dental Surgeon, 7 Balfour Road, Birkenhead (proposed by W. H. Steavenson);

Charles Bousfield Shaw, Member of the Bar, Secretary and Treasurer, Royal Agricultural Benevolent Institution, 8 Milborne Grove, The Boltons, London, S.W. 10 (proposed by J. Evershed); and

A. R. Stoney Smith, White Waltham Grove, near Maidenhead (proposed by R. A. Fisher).

Forty-two presents were announced as having been received since the last Meeting.

Mr. P. J. Melotte and Dr. W. H. Steavenson were appointed Auditors of the Treasurer's accounts for 1920.

A Solution of Ancient Eclipses of the Sun. By J. K. Fotheringham, M.A., D.Litt.

In *M.N.*, **80**, 578–81, I gave the results derived from the times and magnitudes of ancient lunar eclipses, ancient occultations of stars and conjunctions of the Moon with stars, equinox observations, and observations of solar eclipses, for the determination of the accelerations of the Sun and Moon. In the case of the solar eclipses, the result given was based, not on a full discussion, but on a tentative examination made by Herr Schoch of Heidelberg of the effect of introducing into the motions of the Sun and Moon respectively certain corrections which I had suggested in earlier papers. The present paper contains the results of an analysis of those solar eclipses which would appear either from the passages in which they are recorded, or from the investigations of earlier inquirers, to be of value for the determination of the two accelerations.

The eclipses selected for computation are:—

(1) The eclipse of Babylon, − 1062 July 31.
(2) The eponym canon eclipse, − 762 June 15.
(3) The eclipse of Archilochus, − 647 April 6.
(4) The eclipse of Thales, − 584 May 28.
(5) The eclipse of Pindar, − 462 April 30.
(6) The eclipse of Thucydides, − 430 August 3.
(7) The eclipse of Agathocles, − 309 August 15.
(8) The eclipse of Hipparchus, − 128 November 20.
(9) The eclipse of Phlegon, + 29 November 24.
(10) The eclipse of Plutarch, + 71 March 20.
(11) The eclipse of Theon, + 364 June 16.

In each case the day specified is reckoned from midnight.

Other eclipses have been omitted because there is no sufficiently strong presumption as to the magnitude of the eclipse at any particular place. The so-called eclipse of Larissa, which has been variously dated −556 May 19 and −602 May 18, is rejected because the record,* whether true or false, seems clearly to refer to some phenomenon other than an eclipse.

(1) *The eclipse of Babylon.*—L. W. King, *Chronicles concerning Early Babylonian Kings* (1907), **2**, 76: "On the twenty-sixth day of the month Sivan in the seventh year the day was turned to night, and fire in the midst of heaven . . .". The passage is discussed at length by King in the first volume of the same work, pp. 232–240. The eclipse was identified by Dr. Cowell in *M.N.*, **65** (1905), 861–7. The text to which the passage belongs is mainly occupied with portents observed at Babylon, such as wild animals or floods entering the city. There are, however, a few references to other places. Thus mention is made of the Lady of Nineveh, the river Tigris, Bit-Ursag in the district of Nippur, and the city of Kar-bel-matati.† As a rule not more than two or three portents are recorded in one year, and the presumption is that the phenomenon described is a rare one, was observed at Babylon, and may reasonably have been a total eclipse of the Sun. King argues against the identification of the portent with a thunderstorm or dust-storm, and discounts the difference between the phrase "the day was turned to night" and the technical phrase "heaven-darkening," which was from at least the eighth century B.C. used for an eclipse. The portent would appear to have been observed somewhere in the eleventh century B.C., and the eclipse of −1062 July 31 is the only one which can have been total at Babylon within that century. King has a difficulty about the date, because the identification implies that Sivan was falling a month late; but intercalation at Babylon was notoriously irregular, and a latitude of one month in the date of a lunar month is not surprising. A further difficulty, which King does not notice, is that the portent occurred on the 26th day of the month. Unless the Babylonian calendar was in disorder at the time, this is too early for a solar eclipse. The Babylonian months were supposed to begin with the first appearance of the lunar crescent, and on this system −1062 July 31 should have been not the 26th but the 28th day of the month. Herr Schoch has suggested, in an unpublished memoir of which he has kindly sent me a copy, that "26th" is a mistake of the scribe for "28th." Dr. Langdon has kindly verified the reading "26th" on the tablet at the British Museum, but he assures me that the cuneiform symbols for the two numbers are so

* Xenophon, *Anabasis*, **3**, 4, 8: "This city [*i.e.* Larissa] the king of the Persians besieged, what time the Persians strove to snatch their empire from the Medes, but he could in no wise take it; then a cloud hid the face of the sun and blotted out the light thereof, until the inhabitants were gone out of the city, and so it was taken." Xenophon, *March of the Ten Thousand*, tr. by Dakyns (1901), 86.

† Dr. Langdon assures me that Kar-bel-matati was a city in the extreme south of Babylonia.

similar that "26th" might easily have been substituted by an ancient scribe for "28th." In fact, he has gone the length of adopting the identification of this portent with the eclipse of − 1062 July 31 as a starting-point for the construction of Babylonian chronology.

In *Zeitschrift für Assyriologie*, **27** (1912), 388–90, Dr. Weidner rejects the interpretation of the phenomenon as an eclipse on the ground that the technical phrase for an eclipse is not used. He finds references to fire in heaven on a mutilated tablet, on the other side of which there are references to the morning and evening glow of the clouds. He somewhat wildly infers from this that "fire in the midst of heaven," which Dr. Cowell had suggested might mean the corona, refers to a red glow on the clouds, and interprets the whole phenomenon as a thunderstorm. There is, of course, no evidence that the two sides of the tablet which he cites are dealing with cognate subjects.

Father Kugler has kindly answered an inquiry of mine about the phenomenon in question. While rejecting Dr. Weidner's interpretation, he holds that the phenomenon must have been meteorological, not astronomical, because, as he has shown in *Zeitschrift der deutschen morgenländischen Gesellschaft*, **56** (1902), 60–70, the technical term for an eclipse is regularly accompanied by the name of the luminary where an astronomical eclipse is intended, and is constantly used without such a name to indicate a darkness due to meteorological causes. He also urges against the interpretation of the phenomenon as an eclipse that it is recorded as falling on the 26th day of the month. In his opinion the reference is to a shower of meteoric dust, accompanied by glowing meteorites. For examples of such showers he refers to Arago's posthumously published *Astronomie populaire*, **4** (1857), 209–16,* and in particular to the showers of 472 November 5 or 6, 1792 August 27–29, 1814 July 3 and 4, 1819 November. On the last of these occasions Arago asserts that there was an extraordinary darkness of the sky accompanied by fiery apparitions. He cites *Annales de chemie et de physique*, **15** (1820), apparently pp. 422, 426, 427, but only a part of the phenomena which he describes is recorded there, and in particular there is no mention of fiery apparitions, though there is of black rain. It is probable that Arago relied in part on some source to which he left no reference.

Dr. Langdon agrees that "fire in the midst of heaven" should, according to Babylonian usage, be a meteor, but attaches no importance to the fact that the phrase "the day was turned to night," which would excellently describe a total eclipse of the Sun, is not the correct astronomical phrase. He lays stress on the fact that the text in which the passage occurs is not written by an astronomer, but in all probability by a divination scholar; and of course it must be remembered that it is older than those by means of which Father Kugler defines the established usage. It is

* This passage is omitted from the English translation, edited by Smyth and Grant.

certainly remarkable that there should have been an eclipse which may, consistently with other evidence, have been total at or near Babylon at or near the date to which the text refers, and that it should have fallen at the right time of year. I take it accordingly that there is a presumption, but not a certainty, that the eclipse of − 1062 July 31 was total at Babylon, for which I take λ (east longitude) $= 44^\circ{\cdot}50$, ϕ (north latitude) $= 32^\circ{\cdot}52$.

(2) *The eponym canon eclipse.**—The Assyrian Chronicle translated by Dr. Sayce in *Records of the Past*, new series, **2** (1890), 124: "Isid-Raki's-rabe [or rather Ishdi-Sagale †] of the city of Gozan. Insurrection in the city of Assur. In the month Sivan the sun was eclipsed." The date is beyond question, since the Assyrian eponym canon overlaps Ptolemy's Canon of Kings, and the year − 762 is obtained by counting backwards to the year of Ishdi-Sagale. The Assyrian Chronicle is a fuller form of the eponym canon, containing not merely lists of eponyms, but striking events from all parts of Assyria, and Assyrian wars beyond Assyria. The reference to Sivan gives the time of year, and the identification with the eclipse of June 15 in that year, first made by George Smith in 1867,‡ is certain. As the eclipse is the only eclipse mentioned in this Chronicle, which covers an interval of 155 years, there can be no reasonable doubt that it had been reported as a total eclipse. On the other hand, there is no reason why the eclipse should have been total either at Nineveh, where the extant copies of the Chronicle were discovered, or at Calah, which was the Assyrian capital at the date of the eclipse, and where this portion of the Chronicle was probably put together. I have considered it sufficient that the eclipse should have been total somewhere in Assyria, and have taken as the limits between which totality must have occurred, $\lambda 42^\circ{\cdot}19$, $\phi 37^\circ{\cdot}40$ and $\lambda 45^\circ{\cdot}15$, $\phi\ 34^\circ{\cdot}18$.

(3) *The eclipse of Archilochus.*—Archilochus, fr. 74, in Bergk, *Poetæ Elegiaci et Iambographi* (1915): "Nothing there is beyond hope, nothing that can be sworn impossible, nothing wonderful, since Zeus father of the Olympians made night from midday, hiding the light of the shining sun, and sore fear came upon men."

The eclipse was identified by Oppolzer.§ The description clearly refers to a total eclipse, and there is a presumption that Archilochus saw it. As the life of Archilochus was divided between Paros and Thasos, with occasional travels elsewhere, the presumption is that the eclipse was total either at Paros or at Thasos. I find by computation that the width of the belt of totality is practically equal to the distance between these islands, and I have therefore assumed that the eclipse was total somewhere

* For the literature down to 1899 on this and the remaining eclipses, with the exception of (5) and (8), see Professor Ginzel's *Spezieller Kanon der Sonnen- und Mondfinsternisse* (1899).

† Tallqvist, *Assyrian Personal Names* (1914), 104.

‡ See his *Assyrian Discoveries* (1875), pp. 12, 13.

§ *Sitzungsberichte der k. Akad. der Wissenschaften, Vienna, Math.-nat. Classe*, **86** (1882), 790–3.

between the limits $\lambda 24°·56$, $\phi 40°·78$ and $\lambda 25°·25$, $\phi 37°·01$, and have not thought it necessary to exclude any part of the intervening space. The computed time of the eclipse falls about ten in the morning instead of at midday. But a tendency to transfer eclipses to midday will be found again in the cases of the eclipses of Phlegon and Plutarch.

(4) *The eclipse of Thales.*—Herodotus, 1, 74, tr. by Rawlinson (1909), 1, 36, 37: "War broke out between the Lydians and the Medes, and continued for five years, with various success. In the course of it the Medes gained many victories over the Lydians, and the Lydians also gained many victories over the Medes. Among their other battles there was one night engagement. As, however, the balance had not inclined in favour of either nation, another combat took place in the sixth year, in the course of which, just as the battle was growing warm, day was on a sudden changed into night. This event had been foretold by Thales, the Milesian, who forewarned the Ionians of it, fixing for it the very year in which it actually took place. The Medes and Lydians, when they observed the change, ceased fighting, and were alike anxious to have terms of peace agreed on."

An incomplete collection of later references to the eclipse is given by Professor Diels in *Fragmente der Vorsokratiker*, 1 (1912), pp. 8, 9. The war fell, according to Herodotus and several later writers, in the reigns of Alyattes of Lydia and Cyaxares of Media. Cicero, *de Div.*, 1, 49, 112, places the eclipse in the reign of Astyages, the successor of Cyaxares. Eusebius, who separates the eclipse from the war, places both in the reign of Astyages; and Solinus, ed. Mommsen, 85, 9, places the war in the reign of Astyages. Several ancient writers give the date of the eclipse, generally within a few years of −584. Eusebius in his *Chronica* alone gives that precise year. It must be remembered that the eras used by later writers were not adopted till long after the time of Thales, and it was therefore easy for a small error to slip into the conversion of a date. The eclipse was identified with that of −584 May 28 by Bünting, *Chronologia* (1590), 82. But there was formerly a difficulty in the way of accepting this date, because Herodotus appeared to make Astyages die in −557, after a reign of 35 years, and the date −584 seemed in consequence to be inconsistent with Herodotus's statement that the event occurred in the reign of Cyaxares. It is now known, however, from the annalistic tablet of Cyrus that the reign of Astyages lasted till −549, and his accession ought, therefore, to have been somewhere in the neighbourhood of −584. It is impossible to say whether Cyaxares or Astyages was reigning on May 28 of that year, and there appears to be no longer any reason for calling the date in question.

It has frequently been assumed that the battle described by Herodotus must have been in the neighbourhood of the Halys, presumably because Crœsus crossed the Halys when he made war on Persia. But this is quite fanciful. The battle may have been fought almost anywhere in Asia Minor. I have assumed that the eclipse was total somewhere between $\lambda 27^\circ{\cdot}50$, $\phi 37^\circ{\cdot}00$ and $\lambda 38^\circ{\cdot}50$, $\phi 41^\circ{\cdot}00$.

For the method by which Thales predicted this eclipse, see Airy, *M.N.*, **18** (1858), 148, and my paper in the *Journal of Hellenic Studies*, **39** (1919), 181–3.

(5) *The eclipse of Pindar.*—Pindar, Pæan ix., ed. Sandys (1919), p. 546: "Beam of the Sun! O thou that seest afar, what wilt thou be devising? O mother of mine eyes! O star supreme, reft from us (κλεπτόμενον) in the daytime! Why hast thou perplexed the power of man and the way of wisdom, by rushing forth on a darksome track?"

See also Pindar, fr. 142, *ibid.*, p. 576: "God can cause unsullied light to spring out of black night. He can also shroud in a dark cloud of gloom the pure light of day."

The eclipse was identified by Ideler * with that of −462 April 30, which was certainly the largest at Thebes in the time of Pindar. The first of the passages cited is part of a pæan written by Pindar, a Theban, and addressed to the Thebans. The terms used seem to indicate a total eclipse of the Sun, and the second passage, from a dance-song, would appear to refer to the same event. There is a reasonable presumption that the eclipse was total at Thebes, $\lambda 23^\circ{\cdot}32$, $\phi 38^\circ{\cdot}32$, though the present tense, κλεπτόμενον, is consistent with incomplete action.

(6) *The eclipse of Thucydides.*—Thucydides, **2**, 28, tr. by Smith (1919), pp. 309, 311: "During the same summer at the beginning of a lunar month (the only time, it seems, when such an occurrence is possible) the sun was eclipsed after midday; it assumed the shape of a crescent (μηνοειδής) and became full again, and during the eclipse some stars became visible."

The year of this eclipse is clearly defined by Thucydides, and corresponds to our −430. The eclipse was identified by Bünting, p. 108. Though Thucydides does not name the place of observation, it has generally been assumed that it was Athens, an assumption which accords well with the general perspective of Thucydides' narrative of the events of this year. Mr. Munro has, however,† in order to render it possible for Thucydides to have seen stars, and guided by the narrative of negotiations with Thrace, which follows the record of this eclipse, suggested that the real place of observation was at the Odrysian capital in the neighbourhood of Adrianople. In order to examine the question of the visibility of stars, I have computed the stellar magnitudes and altitudes of all the planets and the altitudes of the brighter fixed stars for Athens for the moment of greatest phase of the eclipse, which I have taken to be $5^h\ 36^m$ in the afternoon, local solar time. Treating the stellar magnitudes as valid for stars in the zenith, I have reduced them to the apparent magnitudes proper to their respective altitudes, using $0^m{\cdot}22$ as the coefficient of the atmospheric absorption of light. The resultant magnitudes are Venus −3·35

* See *Pindari Opera*, ed. Boeckh, **2** (2), (1821), 602

† *Classical Quarterly*, **13** (1919), 127, 128.

Mercury +1·3, Mars +4·05, Saturn +0·55, Vega +0·20, α Centauri barely above horizon, Jupiter and Sirius below horizon. It will be seen from this that Vega, the second brightest star above the horizon, was only $\frac{1}{26}$ as bright as Venus. Now, as a general rule, stars other than Venus are not visible to the naked eye during an eclipse except on the verge of totality. In the case of the eclipse of 1912 April 17, I have found no record of any star but Venus being seen without instrumental aid except where the eclipse was total, although there were numerous observers along the long zone where the eclipse passed through France, Belgium, and Germany just missing totality. The eclipse of −430 was an annular eclipse with a maximum phase of about 11·88 digits or 99 per cent. of the Sun's diameter. It fell, therefore, considerably short of totality, and, in view of the faintness of the stars above the horizon, I find it exceedingly difficult to accept Thucydides' statement that some stars became visible. In all probability Venus was the only star visible, and there should have been no difficulty in seeing Venus, even if the magnitude of the eclipse amounted to 10·5 only. As Thucydides mentions the crescent phase of the eclipse, but not the ring phase, it seems probable that the eclipse was not annular at Athens. This is the only presumption that I venture to make, and even this presumption cannot be pressed.

(7) *The eclipse of Agathocles.*—Diodorus, **20**, 5, tr. by Booth (1700), 659: "The next day there was such an Eclipse of the Sun, that the Stars appear'd every where in the Firmament, and the Day was turn'd into Night." This eclipse was first identified by Petavius, *De Doctrina Temporum* (1627), **1**, 804, 805. Since the year is specified by Diodorus, there can be no doubt of the correctness of the identification. As we learn from the context, the eclipse was observed by the fleet of Agathocles the day after it had set out on a voyage lasting six days and six nights, from Syracuse to Latomiæ on the African coast. The historical evidence is insufficient to determine whether he sailed to the north or to the south of Sicily. In calculating the distance traversed it must be remembered that the time of greatest phase is found to be about $7^h\ 40^m$ in the morning, local solar time. I have assumed that the eclipse must have been total somewhere between the limits $\lambda 15°·50$, $\phi 38°·42$, and $\lambda 15°·00$, $\phi 36°·58$. A glance at the chart in Ginzel's *Spezieller Kanon*, p. 186, showing the zone of totality according to different computations, will make it clear that Agathocles is not likely to have travelled further than these points in a direction perpendicular to the line of central eclipse.*

(8) *The eclipse of Hipparchus.*—Cleomedes, *De Motu Circulari Corporum Cœlestium*, **2**, 3: "Once it [*sc.* the Sun] was observed to be totally eclipsed at the Hellespont, when at Alexandria it

* There is a reference to this eclipse in Justin, **22**, 6, but he does not define the magnitude of the eclipse, nor the part of the voyage at which it was observed.

was eclipsed with the exception of one fifth of its own diameter, that is, in appearance [*i.e.* as seen by an observer] with the exception of two digits and a fraction."

Pappus, Commentary on the fifth book of the μεγάλη σύνταξις of Ptolemy*: "For in his first book concerning magnitudes and distances he [*i.e.* Hipparchus] uses the following phenomenon: —an eclipse of the Sun which was actually of the whole Sun in the places round the Hellespont, so that no part of it could be seen at the Moon's edge, while at Alexandria in Egypt the Sun was eclipsed to the extent of about four fifths of his diameter."

No clue is given for the identification of this eclipse apart from the names of Hipparchus and Alexandria, and the magnitudes at the Hellespont and at Alexandria. Herr Schoch in his memoir has discussed the possibility of combining the required phases for the Hellespont and Alexandria, and finds that from -320 to -110 the only eclipses which could be made to give anything approximating to the required combination are the eclipse of -309 August 15, just discussed as that of Agathocles, and the eclipse of -128 November 20, with which Hultsch† has identified the eclipse now in question. No additional eclipse could be brought into the discussion by carrying the investigation back to the foundation of Alexandria in -331. Against the identification of the eclipse of Hipparchus with that of Agathocles Herr Schoch argues that it would have been impossible to judge the magnitude of the eclipse of Agathocles at Alexandria with the naked eye, as that eclipse would reach its greatest phase there about $9^h\ 30^m$ in the morning. The later eclipse, however, would attain its greatest phase not long before sunset, when the disc would be sharply defined and it would be a simple matter to estimate the magnitude. There is also the inherent probability that Hipparchus would collect two observations of an eclipse in his own time more easily than two observations of an eclipse 180 years earlier. I doubt, also, whether an astronomer of -309 would be likely to think of estimating, much less of recording, the magnitude of a solar eclipse. The earliest Greek observation of a lunar eclipse, cited by Ptolemy from Hipparchus, is that of a partial eclipse at Alexandria -200 September 22,‡ where the magnitude is not recorded. From that date to -140 four more lunar eclipses are cited, of which two, in -173 and -140, were partial. In both cases the magnitude is recorded. The Babylonian astronomers of the time of Hipparchus not only observed, but predicted, the magnitudes of lunar eclipses. We should, therefore, expect an attempt to define the magnitude of a solar eclipse in -128, if the Sun were low enough, or the air thick enough, to render it practicable. We should not expect it in -309.

* This Commentary has never been printed. The passage cited and its context are given in Hultsch's paper, "Hipparchos über die Grösse und Entfernung der Sonne," *Berichte der sächsischen Gesellschaft*, **52** (1900), *Phil.-hist. Classe*, 169–200.

† *Loc. cit.*

‡ Ptolemy, *Math. Syn.*, **4**, 11, ed. Heiberg, **1** (1898), 344.

I have, after finding the direction of the zone of totality, assumed as the two limits between which some part of that zone must have been situated $\lambda 26^\circ{\cdot}60$, $\phi 40^\circ{\cdot}40$ and $\lambda 26^\circ{\cdot}30$, $\phi 39^\circ{\cdot}97$. I have also computed the magnitude of the eclipse for Alexandria, and have made the corresponding computations both for the Hellespont and for Alexandria on the less probable assumption that this eclipse is identical with that of Agathocles.

(9) *The eclipse of Phlegon.*—Phlegon, *Olympiades*, fr. 17 (*Rerum naturalium scriptores Græci minores*, ed. Keller, 1877, **1**, 101): "And Phlegon also who compiled the *Olympiads* writes about the same things in his thirteenth book in the following words: 'In the fourth year of the 202nd Olympiad (32–33 A.D.) an eclipse of the Sun took place greater than any previously known, and night came on at the sixth hour of the day, so that stars actually appeared in the sky; and a great earthquake took place in Bithynia and overthrew the greater part of Nicæa.'"

Julius Africanus,* who identifies this eclipse with the darkness at the crucifixion of Christ, attributes to Phlegon the statement that the eclipse took place at full moon. Origen,† however, declares that Phlegon made no such statement, and it will be observed that it is absent from the fragment, cited above, which was extracted from Phlegon by Eusebius.

In spite of the date given by Phlegon, it is impossible to identify this eclipse with any except that of + 29 November 24, which fell in the first, not in the fourth, year of the 202nd Olympiad. The identification was first made by Kepler, *Eclogæ Chronicæ* (1615), 126.‡ Since the zone of totality must have passed through or near Bithynia, it seems reasonable to presume that it was the author's intention to place both the totality and the earthquake at Nicæa, and I have accordingly assumed that the eclipse was total at Nicæa ($\lambda 29^\circ{\cdot}60$, $\phi 40^\circ{\cdot}42$). But I consider the presumption made in the case of this eclipse weaker than any of the other presumptions used in this paper. The total phase would be at the fifth hour rather than the sixth, but, as has been observed, the eclipses of Archilochus and Plutarch are also transferred to midday.

(10) *The eclipse of Plutarch.*—Plutarch, *De Facie in Orbe Lunæ*, 931 D, *Moralia*, ed. Bernardakis, **5** (1893), 434. Plutarch, *Select Essays*, **2** (1918), tr. by Prickard, 282: "Grant me that no one of the phenomena relating to the sun is so like another as an eclipse to a sunset, remembering that recent concurrence of sun and moon, which, beginning just after noon, showed us plainly many stars in all parts of the heavens, and produced a chill in the temperature like that of twilight. If you have forgotten it . . ."

The beginning of Plutarch's dialogue *De Facie* is lost, and in the part preserved to us there is no direct indication of the sup-

* Migne, *Patrologia Græca*, **10** (1857), 89.
† *In Matthæum*, Migne, *Pat. Gr.*, **13** (1862), 1782.
‡ *Opera*, ed. Frisch, **4** (1863), 435.

posed date or place of the dialogue. But something can be learned from the names of the characters. Lamprias, Plutarch's brother, probably elder brother, presided. Like Plutarch, he resided at Chæroneia, nineteen miles from Delphi, and he appears in numerous dialogues, in one of which he is, as here, the narrator. The scene of these is always laid in Greece, Ædepsus, Athens, Chæroneia, Delphi, and Eleusis having their turns. Another character, Theon, is an intimate friend of Plutarch's family, and may therefore be supposed to have lived at Chæroneia. He figures in seven other dialogues, which would all appear to belong either to Delphi or to Chæroneia, with the exception of one which the author may have intended to place at Rome.* Sextius Sylla, the Carthaginian, appears in four other dialogues, at two of which he is the host. In each case the scene is laid at Rome. Lucius, an Etruscan Pythagorean, appears in two other dialogues, as a guest of Sylla at Rome. Apollonides the geometrician, Aristoteles the Peripatetic, Pharnaces the Stoic, and the most famous character of all, Menelaus the astronomer of Alexandria, do not appear in any other dialogue. It seems easier to suppose that the Carthaginian and his Tuscan friend had wandered to Greece than that the Greeks had wandered to Rome. It will be observed that none of Plutarch's Roman friends takes part in the dialogue, and that all except the Carthaginian and the Tuscan have Greek or oriental names. The miscellaneous company would be likely to meet Lamprias and his friend nowhere so easily as at Delphi, visited as it is in other dialogues by travelling men of culture on their way between many different parts of the Roman empire.

The reference to the eclipse is made by Lucius, who is explaining to his audience the points made in a recent discussion at which he and Lamprias had been present, but none of the rest of the company. The leading part in that discussion had been taken by a person who was not present on this occasion, and who is described by more than one member of the company by the title "our comrade." Dr. Hartman † has no doubt that the anonymous comrade was intended to be none other than Plutarch himself. I do not hesitate to accept this opinion. It is difficult to see how the anonymity can otherwise be explained. It is, therefore, highly probable, if not certain, that Plutarch meant the reference to the eclipse to rest on his own authority. The description of the eclipse implies that the company might be expected to remember it, but the speaker seems to assume that some would not. εἰ δὲ μή, translated above, "If you have forgotten it," is of course simply "But, if not." It does not, as Mr. Prickard's translation suggests, imply that those who did not remember the eclipse had forgotten it. It is quite consistent with a supposition that, being at some other part of the world, they had not seen it, or had not seen it as a total eclipse. On the other hand, Lucius would hardly have used the second person plural if he and his Carthaginian friend

* Mestrius Florus is the host, but he may have had a house at Chæroneia.

† *De Plutarcho Scriptore et Philosopho* (1916), 557.

had been the only spectators of the eclipse, and it seems reasonable to suppose that Plutarch meant to imply that the eclipse had been visible to those members of the company who had been at Delphi or Chæroneia at the time. How many of these there were we have no means of saying. The description of the eclipse reads like that of an eye-witness, and I have thought it a reasonable presumption that the eclipse was total either at Delphi or at Chæroneia. "Stars in all parts of the heavens" are not seen except at a total eclipse. Struyck, *Inleiding tot de allgemeene Geographie* (1740), 101,* identified the eclipse with that of 71 March 20, and it is not easy to substitute another identification. The date falls rather earlier than those of most of Plutarch's dialogues. It is true that this identification makes the greatest phase fall about 11 a.m., instead of making the eclipse begin immediately after noon. But, as has been seen, the error in the time is in this respect similar to that made in the cases of the eclipses of Archilochus and Phlegon.

(11) *The eclipse of Theon.*—Theon, εἰς τὸ τοῦ Πτολεμαίου βιβλίον ϛ ὑπόμνημα (1538), 332: ". . . the time reckoned by civil days † and equinoctial hours of the exact ecliptic conjunction which we have discussed, and which took place according to the Egyptian calendar in the 1112th year from the reign of Nabonassar $2\frac{5}{6}$ equinoctial hours after midday on the 24th of Thoth, and according to the Alexandrine calendar reckoned by simple civil days † in the 1112th year of the same reign $2\frac{5}{6}$ equal or equinoctial hours after midday on the 22nd of Payni. . . . And moreover we observed with the greatest certainty the time of the beginning of contact, reckoned by civil and apparent time as $2\frac{5}{6}$ equinoctial hours after midday, and the time of the middle of the eclipse as $3\frac{4}{5}$ hours, and the time of complete restoration as $4\frac{1}{2}$ hours approximately after the said midday of the 22nd of Payni." The date defined by Theon is clearly 364 June 16. This is the only ancient eclipse of the Sun for which an astronomically observed time is recorded. The place, Alexandria, is equally certain. It will be assumed that Theon's times are correct.

The method used in this paper is substantially the same as that used in my paper on "The Eclipse of Hipparchus," *M.N.*, **69** (1909), 204–10. I adopt elements of the motion of Sun and Moon supposed to be approximately correct, and compute the belt of totality or annularity for each eclipse except that of Theon. I make similar computations with two sets of elements in which the secular accelerations of the Moon and Sun respectively have been increased by 1″ per century. For the eclipse of Hipparchus I similarly compute three values for the magnitude at Alexandria, and for the eclipse of Theon three values for the times of beginning, greatest phase, and end of the eclipse at Alexandria. From the belts of totality I compute equations showing the limits between

* Cited by Ginzel, *Spezieller Kanon*, 203.

† νυχθήμερον is properly "a night and a day," *i.e.* a civil or astronomical day, as distinct from a light-day reckoned from sunrise to sunset.

which the two accelerations must lie in order to satisfy the presumptions made in regard to each eclipse. For the magnitude of the eclipse of Hipparchus at Alexandria and for the times of the different phases of the eclipse of Theon, there is one equation for each observation instead of two limiting equations. Graphs representing the different equations are then plotted out on a sheet of paper, and it is thus possible to see to what extent the presumptions are consistent and what values of the two accelerations will satisfy the presumptions which are found to be consistent.

The initial elements adopted for the Sun are those given in Newcomb's Tables of the Sun, reduced to 1800 January 0·0, with the single exception that the coefficient of T^2 in the longitude is taken as $+2''\cdot21$ instead of $+1''\cdot089$. The value adopted corresponds to a sidereal secular acceleration of $+1''\cdot10$.

For the Moon I have assumed a sidereal secular acceleration of $+10''\cdot10$ at epoch 1800 January 0·0. This is equivalent to a correction of $+4''\cdot09$ to Dr. Brown's value. Introducing this as the value for z in Table VI. of my paper on "The Longitude of the Moon," *M.N.*, **80** (1920), 303, I get by interpolation for x and y $-1''\cdot715$ and $+2''\cdot695$ respectively. These are the corrections to Dr. Brown's mean longitude and centennial motion respectively for 1810·0. Reduced to 1800·0 they become $-1''\cdot98$ and $+1''\cdot88$ respectively. Substituting $-1''\cdot98+1''\cdot88T+4''\cdot09T^2$ for the correction to Dr. Brown's mean longitude, reckoned from epoch 1800·0 as given on p. 304 of the paper cited, I obtain a correction of

$$+0''\cdot31-0''\cdot08T-0''\cdot43T^2$$

to the mean longitude of the Moon deduced in that paper. In the final mean longitude deduced on p. 305 of that paper, $25''\cdot22$ is a misprint for $25''\cdot52$. Correcting this misprint and applying the correction just found, I obtain for the preliminary value of the Moon's mean longitude

$$L=335^\circ\ 43'\ 25''\cdot83+\overline{17}32\dot{5}\ 64393''\cdot68T+11''\cdot21T^2+0''\cdot0068T^3,$$

where T is the interval in Julian centuries since 1800 January 0·0.

The elements of the motion of the Moon's perigee and node are taken from Dr. Brown without change, and those of the Sun's motion from Newcomb with the one change specified above. I thus get:—

$$\begin{aligned}
g &= 110^\circ\ 19'\ 38''\cdot02+17179\ 15796''\cdot96T+48''\cdot240T^2+0''\cdot0518T^3\\
F &= 302\ 26\ 57\cdot97+17395\ 27319\cdot85T+\ 3\cdot750T^2-0\cdot0012T^3\\
L' &= 279\ 54\ 49\cdot33+\ 1296\ 02765.95T+\ 2\cdot210T^2\\
g' &= \ \ 0\ 24\ 41\cdot53+\ 1295\ 96580\cdot14T+\ 0\cdot616T^2-0\cdot0120T^3
\end{aligned}$$

where g and g' are the mean anomalies of the Moon and Sun respectively, F is the Moon's argument of latitude, and L′ is the mean longitude of the Sun.

The mean obliquity of the ecliptic is taken from my paper on "The Secular Acceleration of the Sun as determined from Hipparchus' Equinox Observations," *M.N.*, **78** (1918), 411, and is expressed by the formula

$$\epsilon = 23^\circ\ 27' 31''\cdot 85 - 47''\cdot 090\mathrm{T} - 0''\cdot 009\mathrm{T}^2 + 0''\cdot 0018\mathrm{T}^3,$$

where T is the number of solar centuries elapsed since 1850·0.

I call these elements Set A.

A second set of elements was, as explained above, based on the assumption that the preliminary value of the Moon's secular acceleration was to be increased by $+1''$ per century. In order to make this correction harmonise with the modern observations, a further correction of $-0''\cdot 77 + 0''\cdot 41\mathrm{T}$ has to be applied to the Moon's longitude at epoch 1810·0.* The total correction reduced to epoch 1800·0 becomes

$$-0''\cdot 80 + 0''\cdot 21\mathrm{T} + 1''\cdot 00\mathrm{T}^2.$$

This correction has to be applied to the values given above for L, g, and F. The other elements remain unchanged.

The elements as thus corrected are called Set B.

The third set of elements is obtained by applying a correction of $+1''$ per century to the secular acceleration of the Sun as assumed in Set A. I have no such elaborate discussion of modern observations of the Sun as would enable me to postulate the resulting corrections to the epoch and modern centennial motion of the Sun. The total correction to be applied to the Sun's longitude at epoch 1800·0 is therefore

$$+1''\cdot 00\mathrm{T}^2.$$

This correction has to be applied to the values assumed in Set A for L′ and g'. The other elements of Set A remain unchanged.

The elements thus corrected are called Set C.

The computation with these three sets of amended elements was made with the aid of syzygy tables constructed by Herr Schoch, which to a large extent supersede Oppolzer's *Syzygien-Tafeln*.† Herr Schoch's tables have never been published, but he has been kind enough to supply me with a copy. In these tables the secular terms and secular alterations in the coefficients of periodic terms are more accurately represented than in Oppolzer's tables, and Newcomb's value for the eccentricity of the Earth's orbit with its secular changes takes the place of Leverrier's.

Herr Schoch has also diminished Oppolzer's value for the ratio which the radius of the Moon's figure bears to the Earth's equatorial radius. Oppolzer, following Hansen, had taken this as 0·27296. Herr Schoch has adopted 0·27252, which he believes to be correct. For occultations the *American Ephemeris* now uses

* See Table VI. in my paper in *M.N.*, **80**, 303.

† *Publication der astronomischen Gesellschaft*, **16** (1881).

Hansen's value for this quantity when the effect of irradiation is included, and the same value as Herr Schoch when the effect of irradiation is ignored. The value now used for eclipses in the ephemerides is 0·27227. The effect of using too large a value for this quantity is to widen belts of totality and to narrow belts of annularity. In view of the omission of small terms and the possibility of errors in some of the other terms, it is perhaps safer to allow a little margin and to adopt slightly broader belts of totality than the most modern figures would give. This is particularly the case in the present paper, where the eclipses are with one exception total. I have, therefore, thought it sufficient to accept Herr Schoch's correction of this term. As the term in question forms a part of the expression for the radius of the penumbra on the fundamental plane, expressed in terms of the Earth's equatorial radius, Oppolzer's u_a', Herr Schoch's tables apply a constant correction of $-0·00044$ to that term in addition to the corrections arising from changes in the arguments of the periodic terms. The same correction has to be applied to the radius of the umbra on the same plane, Oppolzer's u_i'. This is best done by applying the correction $-0·00088$ to the constant in Oppolzer's formula, $u_i'=0·5473-u_a'$. We thus have $u_i'=0·5464-u_a'$.

The computation of the belts of totality or annularity proceeds by the formulæ given in Oppolzer's *Syzygien-Tafeln*, 20, 21, [53], and repeated in his *Canon der Finsternisse*,* IX, XXIII. A few simplifications have been introduced which do not affect the accuracy of the result within the limits attempted, and in addition to the inequalities used by Oppolzer, the long-period term $-0^{d}·00033 \sin(212°·4+132°·92T)$ has been introduced into the time of each conjunction, and the long-period term $+0^{d}·000165 \sin(211°·0+20°·20T)$ into the time of conjunction for the eclipses of Phlegon and Plutarch. A correction of $+0°·001$ has been applied to all the terrestrial latitudes determined, in order to make them agree with Dr. Brown's value for the ellipticity of the Earth's figure, viz. 1/294. The latitudes on the northern and southern limits of each belt corresponding to the presumed longitudes were found by interpolation, except in the case of the eclipse of Plutarch, where, owing to the direction of the belt, it was thought better to find the longitudes corresponding to the presumed latitudes. A comparison of the results obtained by the different sets of elements showed what proportion of the difference between the elements was permissible consistently with the original presumptions concerning the eclipses. The conclusion from each eclipse is expressed by means of two linear equations, which, when represented by graphs on a diagram, show the limits within which the two accelerations must fall.

For the eclipse of Hipparchus the magnitude at Alexandria is computed by the formulæ given in Oppolzer's *Canon der Finsternisse*,

* *Denkschriften der kaiserlichen Akademie der Wissenschaften, math.-naturw. Classe*, 52 (Vienna, 1887).

XXIV, XXV, except that for the final equation I have substituted a more accurate one:—

$$\text{Greatest phase} = 12\frac{u_a - m}{u_a - u_i},$$

where $u_a = u_a' - \eta f_a \sin\delta' - \xi f_a \cos\delta' \cos t,$

$$u_i = u_i' - \eta f_i \sin\delta' - \xi f_i \cos\delta' \cos t,$$

the symbols on the right-hand side of each equation having the same meaning as in Oppolzer. The effect of the alterations in the elements made in Sets B and C is determined as before, and an equation is found showing values for the two accelerations satisfying the recorded magnitude. The magnitude of the eclipse of Thucydides at Athens is determined in the same manner.

For the eclipse of Theon the time of greatest phase is found by the formulæ given in Oppolzer's *Canon*, XXIV, and the times of beginning and ending by the formulæ on p. XXVIII of the same work.

The computation of the corrections to be applied to Herr Schoch's tables to make them accord with the elements of Sets A, B, C and the computation of the belts of totality or annularity for each of the first ten eclipses were duplicated by Herr Schoch, whose assistance I have found of the greatest value throughout.* Table I. gives some of the principal stages in each computation. For the meanings of the symbols used see Oppolzer's *Canon*, VIII, IX. Table II. gives in the case of eclipses (2), (3), (4), (7), (8) the points where the southern boundary of the belt of totality crosses the meridian on which the northern extreme permitted by the historical evidence as discussed above is situated, and the point where the northern boundary of the belt of totality crosses the meridian on which the southern extreme permitted by the historical evidence is situated. Where, as in the cases of (1), (5), (9), a definite point appears to be indicated by the historical evidence, the intersection of both limits of the belt of totality with the meridian passing through that point is shown. For (6) the intersection of the southern boundary of the belt of annular eclipse with the meridian of Athens is shown. For (10) the table gives the intersection of the southern boundary of the belt of totality with the parallel of latitude of Delphi, and the intersection of the northern boundary of the belt of totality with the parallel of latitude of Chæroneia. According to the values used in 4C, the total phase in longitude 38°·50 falls after sunset.

The linear equations for the two accelerations were then determined as follows:—Let γ, δ be the given latitudes corresponding respectively to the two given longitudes α, β for an eclipse. Let γ_1, γ_2, γ_3 be the latitudes computed above, corresponding to longitude α according to the three sets of elements A, B, C, and

* Herr Schoch has not duplicated my computation of the eclipse of −309 for the Hellespont.

TABLE I.

No.	Date.	G.M.T. of Conjunction. h m	L'.	Z.	ε.	P.	Q.	log p.	log ΔL.	log q.	u a.	log fa.	μ.	γ.
1A	−1062 July 31	5 29·9	116°·564	+0°·12	23°·826	178°·150	176°·13	0·7005	9·7528	8·7490	0·5391	7·6669	262°·78	+0·1612
1B	,,	5 3·4	116·547	,,	,,	178·132	176·11	,,	,,	,,	,,	,,	256·15	+0·1628
1C	,,	5 55·1	116·804	+0·13	,,	178·394	176·36	0·7006	,,	,,	,,	,,	269·00	+0·1399
2A	−762 June 15	8 21·1	74·761	−2·09	23·789	176·773	174·93	0·6988	9·7554	8·7509	0·5353	7·6627	308·07	+0·2800
2B	,,	8 0·0	74·747	,,	,,	176·759	174·92	,,	,,	,,	,,	,,	302·81	+0·2812
2C	,,	8 40·4	74·949	−2·07	,,	176·963	175·12	0·6989	9·7553	8·7508	,,	,,	312·84	+0·2636
3A	−647 Apr. 6	8 34·2	9·080	+0·71	23·774	171·798	172·44	0·6909	9·7638	8·7594	0·5319	7·6649	309·61	+0·6968
3B	,,	8 14·9	9·067	,,	,,	171·784	172·42	,,	,,	,,	,,	,,	304·78	+0·6979
3C	,,	8 51·9	9·255	+0·70	,,	171·972	172·60	,,	,,	8·7595	,,	,,	313·99	+0·6822
4A	−584 May 28	14 34·4	59·713	−2·33	23·770	3·682	4·20	0·6906	9·7640	8·7590	0·5303	7·6623	40·13	+0·3134
4B	,,	14 15·9	59·700	,,	,,	3·669	4·19	,,	,,	,,	,,	,,	35·52	+0·3123
4C	,,	14 50·5	59·876	,,	,,	3·846	4·36	,,	,,	,,	,,	,,	44·12	+0·3273
5A	−462 Apr. 30	12 1·6	33·404	−1·38	23·752	175·835	174·21	0·6971	9·7575	8·7529	0·5345	7·6632	2·70	+0·3598
5B	,,	11 45·3	33·394	,,	,,	175·823	174·19	,,	,,	,,	,,	,,	358·64	+0·3608
5C	,,	12 16·7	33·553	−1·41	,,	175·983	174·35	0·6972	9·7574	8·7528	0·5346	,,	6·48	+0·3471
6A	−430 Aug. 3	14 51·4	124·629	+0·53	,,	170·757	173·14	0·7232	9·7277	8·7263	0·5531	7·6665	44·65	+0·8450
6B	,,	14 35·2	124·618	,,	,,	170·745	173·13	,,	,,	,,	,,	,,	40·62	+0·8463

9

TABLE I.—*continued.*

No.	Date.	G.M.T. of Conjunction.	L′.	Z.	ε.	P.	Q.	log p.	log ΔL.	log q.	u′a.	log fa.	μ.	γ.
		h m	°	°	°	°	°						°	
6C	− 430 Aug. 3	15 7·6	124·775	+0·53	23·752	170·903	173·29	0·7232	9·7278	8·7263	0·5530	7·6665	48·68	+0·8320
7A	− 309 Aug. 15	8 6·7	136·698	,,	23·733	3·896	2·69	0·6928	9·7612	8·7574	0·5350	7·6679	300·30	+0·3333
7B	,,	7 52·3	136·689	,,	,,	3·887	2·68	,,	,,	,,	,,	,,	296·70	+0·3325
7C	,,	8 20·1	136·831	,,	,,	4·029	2·80	,,	9·7613	,,	0·5349	,,	303·62	+0·3446
8A	− 128 Nov. 20	13 7·2	236·193	−2·76	23·711	9·545	11·53	0·7001	9·7518	8·7498	0·5454	7·6775	17·44	+0·8271
8B	,,	12 55·3	236·184	,,	,,	9·536	11·52	,,	,,	,,	,,	,,	14·45	+0·8264
8C	,,	13 19·2	236·308	,,	,,	9·659	11·64	0·7000	9·7519	8·7499	,,	,,	20·40	+0·8368
9A	+ 29 Nov. 24	9 24·5	240·975	−2·50	23·693	171·338	169·56	0·6969	9·7550	8·7526	0·5437	,,	325·53	+0·7457
9B	,,	9 14·4	240·967	,,	,,	171·330	169·55	,,	,,	,,	,,	,,	323·02	+0·7464
9C	,,	9 34·6	241·072	−2·49	,,	171·436	169·66	0·6970	9·7549	,,	,,	,,	328·02	+0·7375
10A	+ 71 Mar. 20	9 30·5	357·578	+2·08	23·690	7·256	9·69	0·7138	9·7380	8·7354	0·5479	7·6678	318·81	+0·6503
10B	,,	9 21·0	357·571	,,	,,	7·249	9·68	,,	,,	,,	,,	,,	316·44	+0·6497
10C	,,	9 40·0	357·666	,,	,,	7·346	9·78	0·7137	9·7381	,,	0·5478	,,	321·17	+0·6582
11A	+ 364 June 16	12 54·7	85·110	−0·83	23·651	6·437	8·28	0·6991	9·7551	8·7505	0·5353	7·6625	13·08	+0·5581
11B	,,	12 48·1	85·106	,,	,,	6·432	8·27	,,	,,	,,	,,	,,	11·42	+0·5576
11C	,,	1 0·9	85·169	−0·84	,,	6·495	8·34	0·6990	,,	8·7506	,,	,,	14·62	+0·5630

TABLE II.

	Longitude.	Latitude.	Longitude.	Latitude.		Longitude.	Latitude.	Longitude.	Latitude.
1A	44°·50	31°·30	44°·50	32°·43	6A	23°·72	47°·85	°	°
1B	,,	29·35	,,	30·45	6B	,,	52·14		
1C	,,	31·69	,,	32·85	6C	,,	42·07		
2A	42·19	37·61	45·15	39·89	7A	15·50	36·99	15·00	38·66
2B	,,	36·72	,,	39·20	7B	,,	36·32	,,	37·95
2C	,,	37·23	,,	39·28	7C	,,	38·21	,,	39·91
3A	24·56	39·17	25·25	43·08	8A	26·60	40·27	26·30	40·55*
3B	,,	36·73	,,	40·57	8B	,,	39·21	,,	39·49*
3C	,,	40·21	,,	44·03	8C	,,	42·38	,,	42·57*
4A	38·50	33·48	27·50	39·69	9A	29·60	39·41	29·60	40·44
4B	,,	35·04	,,	41·10	9B	,,	41·51	,,	42·49
4C	,,	[32·80]	,,	39·27	9C	,,	36·50	,,	37·56
5A	23·32	39·56	23·32	41·40	10A	22·78	38·48	22·38	38·50
5B	,,	39·33	,,	41·20	10B	25·22	38·48	24·82	38·50
5C	,,	38·92	,,	40·72	10C	19·56	38·48	19·14	38·50

let δ_1, δ_2, δ_3 be the computed latitudes corresponding similarly to longitude β. Put L = secular acceleration of the Moon, and L′ = secular acceleration of the Sun. Then the limiting values of L and L′ are given by the two equations—

$$L' + \frac{\gamma_1 - \gamma_2}{\gamma_1 - \gamma_3} L = \frac{\gamma_1 - \gamma + 10''\cdot 10(\gamma_1 - \gamma_2)}{\gamma_1 - \gamma_3} + 1''\cdot 10,$$

$$L' + \frac{\delta_1 - \delta_2}{\delta_1 - \delta_3} L = \frac{\delta_1 - \delta + 10''\cdot 10(\delta_1 - \delta_2)}{\delta_1 - \delta_3} + 1''\cdot 10.$$

Where the computation has been made for one meridian only, δ will be $=\gamma$. Where, as in (10), the computation has been made for given parallels of latitude instead of given meridians, α and β must represent the given latitudes, and γ, γ_1, γ_2, γ_3, δ, δ_1, δ_2, δ_3 must be taken to be longitudes.

The resulting equations are :—

[TABLE

* If the eclipse of Hipparchus be identified with that of Agathocles, these figures should be 40°·12, 39°·71, 41°·07. These figures should be compared with the figures in the two left-hand columns for 7A, 7B, 7C.

TABLE III.

(1) Upper limit	$L' - 5{\cdot}00L = -46''{\cdot}27$	(6)	$L' - 0{\cdot}74L = -4''{\cdot}66$
Lower ,,	$L' - 4{\cdot}71L = -46{\cdot}26$	*(7) Upper limit	$L' - 0{\cdot}55L = -3{\cdot}27$
(2) Upper ,,	$L' + 1{\cdot}13L = +21{\cdot}87$	Lower ,,	$L' - 0{\cdot}57L = -6{\cdot}32$
Lower ,,	$L' + 2{\cdot}34L = +25{\cdot}28$	(8) Upper ,,	$L' - 0{\cdot}50L = -3{\cdot}89$
(3) Upper ,,	$L' - 2{\cdot}35L = -21{\cdot}05$	Lower ,,	$L' - 0{\cdot}52L = -4{\cdot}44$
Lower ,,	$L' - 2{\cdot}62L = -31{\cdot}71$	(9) Upper ,,	$L' - 0{\cdot}71L = -6{\cdot}06$
(4) Upper ,,	$L' - 3{\cdot}36L = -26{\cdot}44$	Lower ,,	$L' - 0{\cdot}72L = -6{\cdot}52$
Lower ,,	$L' - 2{\cdot}29L = -33{\cdot}09$	(10) Upper ,,	$L' - 0{\cdot}76L = -6{\cdot}49$
(5) Upper ,,	$L' + 0{\cdot}29L = +8{\cdot}56$	Lower ,,	$L' - 0{\cdot}75L = -6{\cdot}65$
Lower ,,	$L' + 0{\cdot}36L = +6{\cdot}68$		

The words "upper" and "lower" in this table refer to the position of the graphs on the accompanying diagram.

The magnitudes of the eclipse of Thucydides at Athens and of Hipparchus at Alexandria are found to be as follows. They are expressed in digits, *i.e.* in twelfths of the diameter.

	A.	B.	C.
(6)	10·53	10·10	11·21
†(8)	9·43	9·75	8·92

As the recorded magnitude of the latter is 9·6, the resulting equation is

$$L' - 0{\cdot}62L = -5''{\cdot}48.\ddagger$$

For the eclipse of Theon the hour angles of the different phases are as follows:—

	A.	B.	C.	Observed.
Beginning	52°·30	50°·32	54°·26	42°·50
Greatest phase	65·66	63·62	67·53	57·00
End	77·86	75·78	79·65	67·50

The resulting equations are:—

(11)	Beginning	$L' - 1{\cdot}01L = -14''{\cdot}10$
	Greatest phase	$L' - 1{\cdot}09L = -15{\cdot}08$
	End	$L' - 1{\cdot}16L = -16{\cdot}42$

The graphs representing the equations show at a glance how

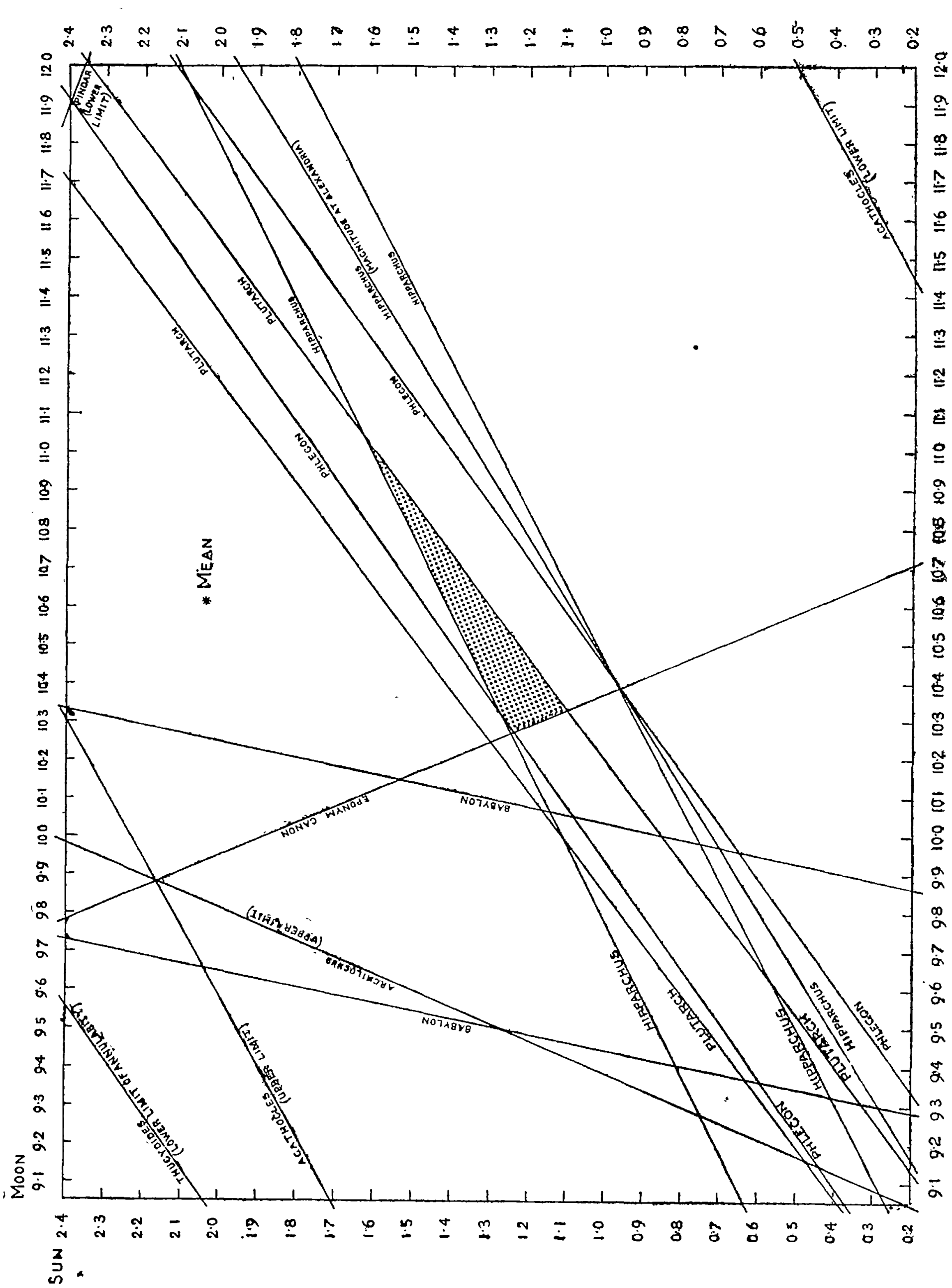
* If the eclipses of Agathocles and Hipparchus be identified, the equations for (7) and (8) combined are:—

Upper limit $L' - 0{\cdot}55L = -3''{\cdot}27$
Lower ,, $L' - 0{\cdot}43L = -3''{\cdot}42$

† These figures should be 9·14, 9·29, 8·82 if the eclipse be identified with that of Agathocles.

‡ $L' - 0{\cdot}44L = -4''{\cdot}75$ if the eclipse be identified with that of Agathocles.

far the initial presumptions made in this paper are consistent. All values represented on the diagram fall between the two limiting equations for the eclipse of Thales. On the other hand, the equations resulting from the eclipse of Theon fall entirely outside



the diagram. They only confirm what is clear from the discordances in the times of occultations, etc., and in the times of lunar eclipses, that the ancients had great difficulty in determining the time. If the final values determined in this paper are correct, Theon's time was 37, 32, and 38 minutes slow at the three observed phases of the eclipse. He would appear to have observed the intervals with some approach to precision, but to have made a large initial error in the time.

It would appear that the presumption that the eclipse of Pindar was total at Thebes is not justified. The eclipse of Thucydides cannot have been annular at Athens, but the record seems to imply that it was only partial. Nos. (3), (7), (8), (9), (10) are found to be perfectly consistent, but it is impossible to reconcile with them both the eclipse of Babylon and the eponym canon eclipse, if we assume, as has been done in this paper, that the former was total in Babylon and the latter somewhere in Assyria. Now it seems easier to suppose that the phenomenon recorded in the Babylonian chronicle was something other than an eclipse, or, if an eclipse, was total in southern Babylonia and not at Babylon itself, than to suppose that the eponym canon eclipse was total in no part of Assyria. That chronicle does not record events outside Assyria except where the Assyrians were fighting, and it is difficult to see why an eclipse which was only known as partial should be recorded at all.

I take it, then, that the evidence of the ancient eclipses of the Sun is best satisfied by values falling within the triangle shaded on the diagram. This triangle is defined by the eponym canon eclipse and those of Hipparchus and Plutarch. It would be difficult to shake the presumptions made concerning the eclipse of Hipparchus, but those made concerning the eponym canon eclipse and the eclipse of Plutarch fall short of certainty, and, if either of these is abandoned, a larger area on the diagram becomes available. I have thought it desirable, therefore, to compare the result deduced from the solar eclipses with those deduced from four other classes of evidence and set forth in my paper in *M.N.*, 80, 581.

Using L for the lunar acceleration, L′ for the solar, and D for the difference, the following table exhibits the values derived from different classes of observations:—

	Lunar Eclipse Times.	Lunar Eclipse Magnitudes.	Occultations, etc.	Equinoxes.
L	...	...	$+10''\cdot8\pm0''\cdot70$	...
L′	...	$+1''\cdot78\pm0''\cdot45$	...	$+1''\cdot93\pm0''\cdot27$
$D=L-L'$	$+7''\cdot9\pm1''\cdot3$			

The weighted means derived from these four classes are $L=+10''\cdot61$, $L'=+2''\cdot04$, where L′ comes out with a weight seven times as great as that attaching to L. I have shown this mean on the diagram. It is clear that the apex of the triangle shaded in the diagram would come nearer to this mean than any point within the triangle. This would give $L=+11''\cdot0$, $L'=+1''\cdot6$. It is, perhaps, an extreme course to push both the eclipse of Plutarch and the eclipse of Hipparchus to the extreme limit of permissible values, and I therefore propose to adopt $L=+10''\cdot8$, $L'=+1''\cdot5$ as the best values attainable on the evidence before us. The nearer to the apex we fix our values, the further we bring the eponym canon eclipse into Assyria, and the nearer we bring the eclipse of Pindar to totality at Thebes.

With these values I find that the belt of totality of the eclipse of − 1062 July 31 should have passed through southern Babylonia, covering the cities of Eridu, Lagash, Larsa, Ur, and Karbel-matati, and the great astronomical centre of Erech. I find that its greatest phase at Babylon should have been at $6^h\ 44^m$ a.m. local solar time, and that its magnitude at Babylon should have been 11·54. If the phenomenon described is an eclipse, this would appear to satisfy the record sufficiently.*

If we adopt these values, the value derived from the occultations is confirmed absolutely, the lunar magnitudes are satisfied within 62 per cent., the lunar eclipse times within 108 per cent., and the equinoxes within 159 per cent. of their respective probable errors. This is a higher standard of agreement than the theory of probability would lead us to expect, and the mutual agreement of these five lines of inquiry is at once a confirmation of the value of each class of evidence as treated in the discussions in which the conclusions were drawn, and a reason for believing that the values now given cannot be in error by more than a small fraction of a second.

The substitution of $+10''\cdot8$ for $+10''\cdot53$ as the value of the lunar acceleration involves some slight modifications in the other constants of the Moon's longitude given in my paper in *M.N.*, **80**, 305. Interpolating in Table VI. of that paper and reducing to epoch 1800·0, I obtain for the mean longitude of the Moon and the great empirical term

$$335^\circ\ 43'\ 25''\cdot26 + 1336^r\ 307^\circ\ 53'\ 13''\cdot82T + 11''\cdot91T^2 + 0''\cdot0068T^3$$
$$+ 13''\cdot60 \sin (104^\circ\cdot2 + 139^\circ\ T),$$

where T is the interval in Julian centuries since 1800 January 0·0, G.M.T.

It may be well to repeat the caution that the precise values given for the two accelerations represent only the mean accelerations between the ancient observations, which fall mainly between − 300 and + 150, and the modern observations of precision which begin about 1658. The mean motion adopted for modern times is based on the assumption that only one empirical term of very long period is to be used, and the term actually used has a period of 258 years, so that its mean value through the range of modern observations is zero. By adopting two or more empirical terms of long period it would be possible to obtain a different mean motion for modern times and therefore a different acceleration, but in the present state of our knowledge such a mean motion would be a mere mathematical contrivance. The mean acceleration found

* The proposed values satisfy both identifications of the eclipse of Hipparchus for the Hellespont and give 9·5 as the magnitude at Alexandria if the eclipse of − 128 is taken as that of Hipparchus, 9·1 if that of − 309 is adopted. If the latter was total anywhere on the Hellespont, its magnitude at Alexandria cannot have exceeded 9·2.

in this paper is the actual mean acceleration of the last two thousand years. It may or may not have been subject to fluctuations.*

It may also be observed that, although the solar eclipses give no indication of the acceleration of the Sun which could not equally be explained by a negative acceleration of the Moon's node, the agreement with the ancient occultations and equinox observations which are independent of the position of the node shows that there is no room left for any addition to the theoretical acceleration of the node, and I am thus able to confirm the conclusion which I announced in *M.N.*, 78, 423, that the agreement between the observed and theoretical motions of the Moon's node has held good for the last two thousand years.

I repeat my main conclusion :—

All classes of ancient observations, including the solar eclipses discussed in this paper, are adequately satisfied by the following accelerations :—

Sidereal secular acceleration of the Moon = + 10″·8 per century.
" " " " Sun = + 1″·5 "

University Observatory, Oxford:
1920 *December* 1.

Observations of Saturn 1920 *November* 6 to *November* 20. By P. H. Hepburn, M. A. Ainslie, W. H. Steavenson, and R. L. Waterfield.

According to the elements of H. Struve, the date of passage of the earth through the ring-plane of Saturn was 1920 November, $7^d\ 5^h$ G.M.T. (about), after which date the unilluminated face of the ring would be presented to the earth.

By kind permission of the Astronomer Royal (to whom the authors desire to express their indebtedness), the following observations were made with the 28-inch refractor of the Royal Observatory, Greenwich. The power used was 450 in each case.

November 6, $17^h\ 00^m$ to $17^h\ 10^m$ (approx.). Observers P. H. H. and W. H. S. Seeing very poor, with gathering clouds which wholly obscured the planet after a few minutes' observation. The ring was conspicuous, and, so far as could be judged under the adverse conditions, the ansæ were continuous and regular.

November 7, $16^h\ 30^m$ to $18^h\ 00^m$ (approx.). Observer M. A. A. Clear sky, seeing pretty good, occasionally very good. Not the slightest trace of ring seen. The shadow of the ring on the planet was well defined; sharp and black to south, fuzzy to north, the edge on the latter side being distinctly concave. In the centre the breadth of the shadow was estimated as being about twice the

* The values here proposed satisfy the eclipse of + 1241 October 6, placing the belt of totality wholly within the well defined, but somewhat wider, zone within which the records make it clear that stars were visible. But that eclipse is satisfied by most of the values that have been proposed.